Computational Studies of Narrow Electron Beams And Oblique Inertial Alfven Waves In a Gravitationally Bound Density Gradient

Defense Presentation
Lukas Mandrake
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Computational Studies of Narrow Electron Beams and Oblique Inertial Alfven Waves In a Gravitationally Bound Density Gradient

Abstract of Work

Investigation 1: Beam-Plasma Structure

- Novel gradient-supporting electrostatic PIC simulation
- Launch finite (8.4 λ_{De}), cold, medium-velocity (4 v_{Te}) e^- beam into gradient
- “Fast solitary waves” form (e^- phase space holes)
- Complex interactions with gradient affect structure speed and distribution

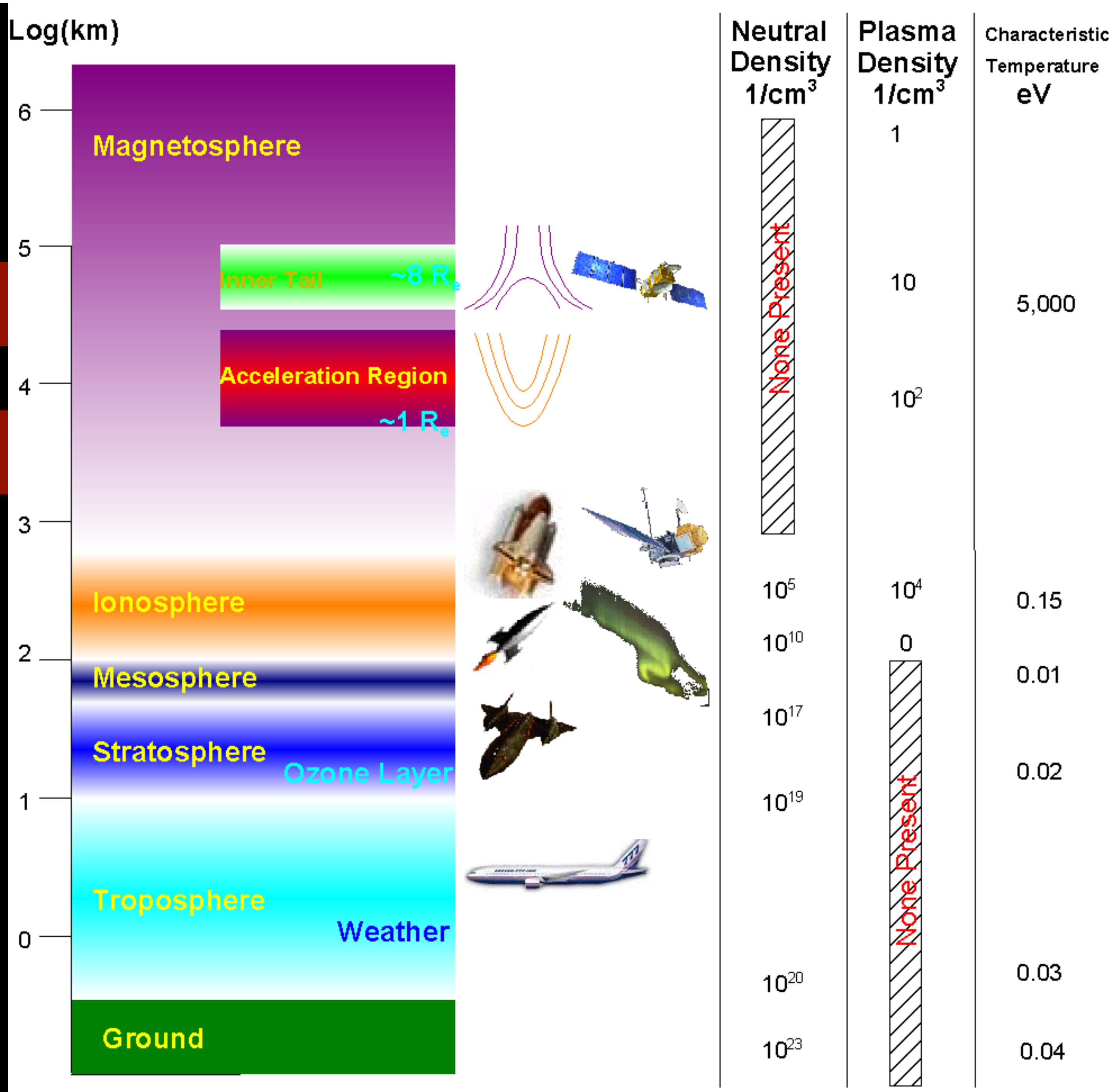
Investigation 2: Oblique Inertial Alfven Waves

- Electromagnetic extension of above PIC code using Darwin approximation
- Identify and overcome nine system requirements for Alfven propagation
- Alfven-like wave demonstrates wave reflection and λ modification
- System memory limitations prevent completion of work

Auroral Review

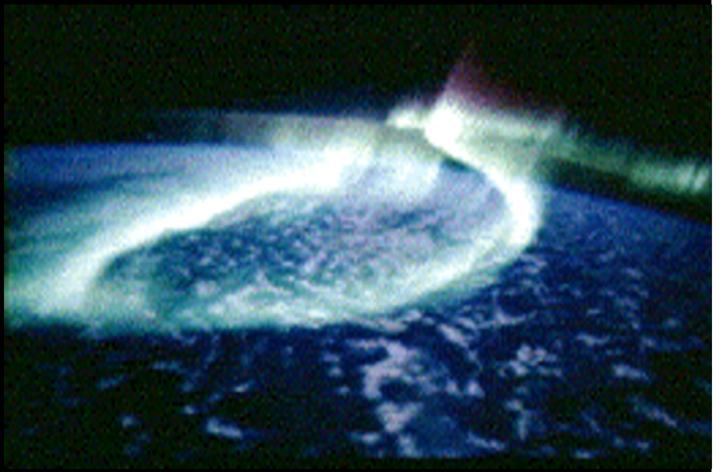
Investigation 2: Inertial Alfven Waves (4-6 R_E)

Investigation 1: Solitary Structures (2000-4000 km)



Auroral Curtain Scales

- Curtains are very tall (500 km) and typically 200 km above ground
- Can range in width from 100 m ($1\lambda_{De}$) discrete aurora scale to 50 km ($500\ \lambda_{De}$) inverted V scale



Sailing upside down, 115 nautical miles above Earth, the crew of the Space Shuttle Endeavor made these spectacular time exposure of the southern aurora (aurora australis) in October of 1994

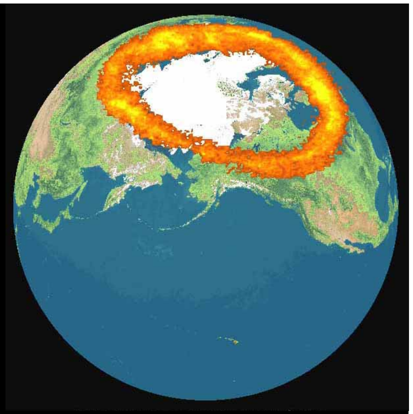
Both pictures are taken from the NASA shuttle

Discrete Aurora (Thin Arcs)

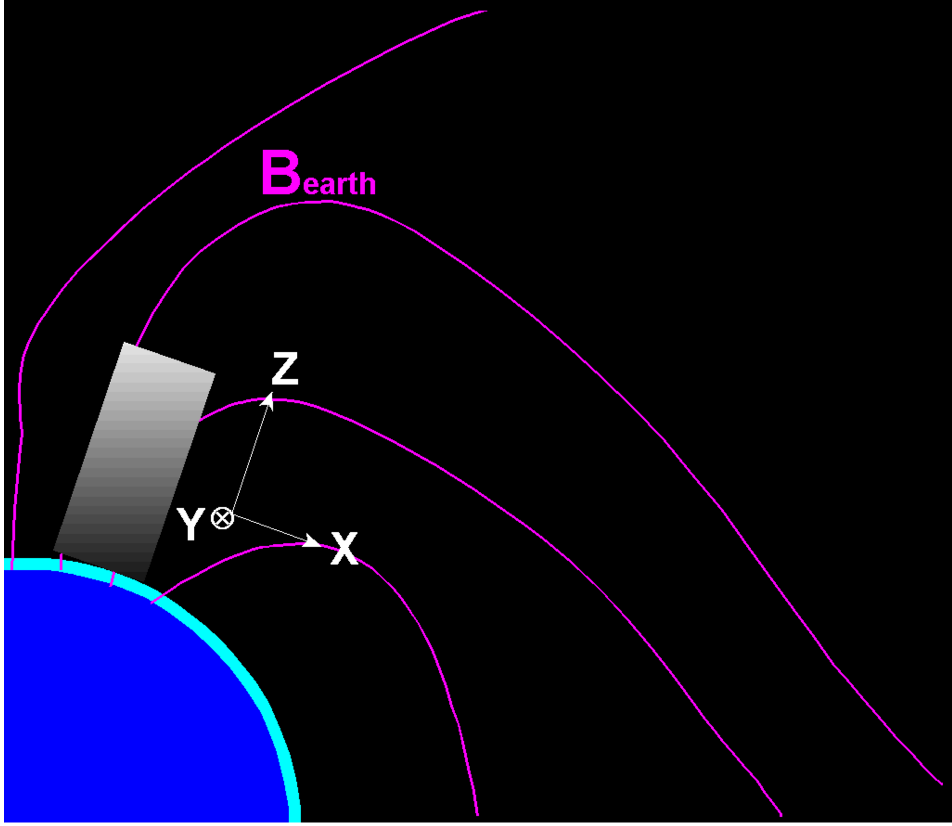
- Discrete aurora occur nested within quiet inverted V arcs [Akasofu 74]
- Surprisingly thin (100 m ~ 1 magnetospheric λ_{De} or less)
- Multiple discrete aurora are often present
- Suggest research in narrow electron beams and small perpendicular wavelength Alfvén waves are directly applicable to auroral generation mechanism

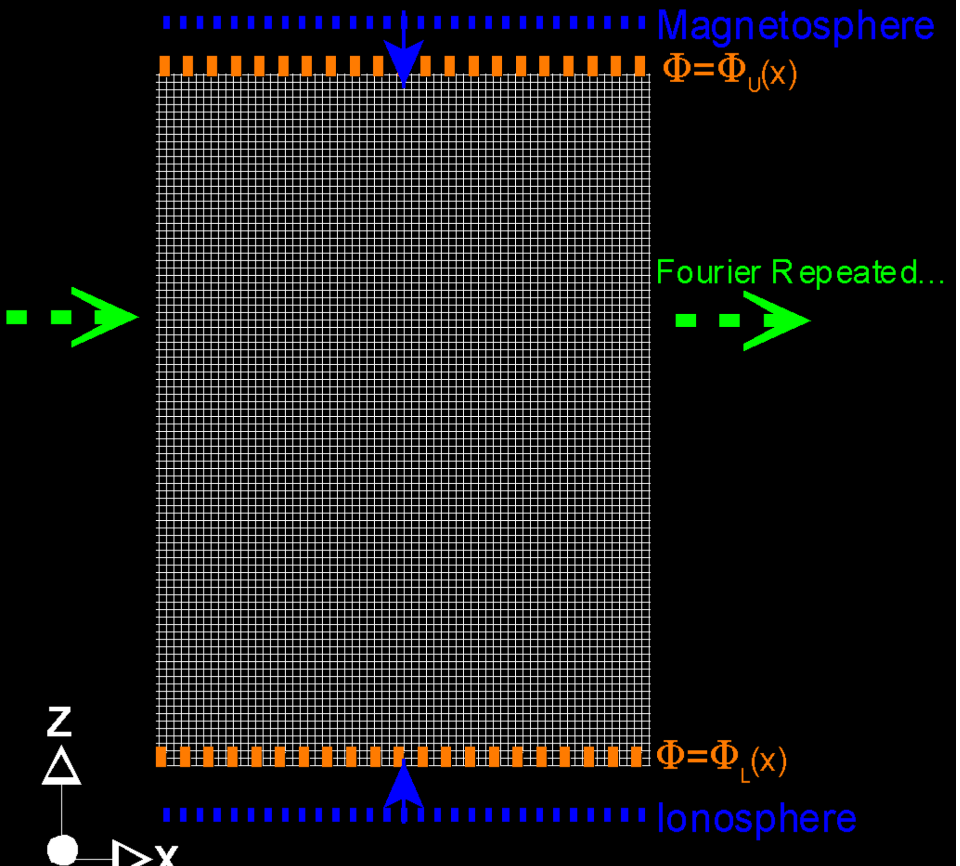
General Simulation Summary

Simulation Coordinates



- Oval curtain extends along suppressed simulation dimension
- x periodicity represents multiple oval curtain systems
- Magnetic gradient ignored due to periodicity concerns
- Box boundaries represent magnetosphere and ionosphere
- Φ set to zero along both boundaries for stability reasons





Code Details

System Configuration

- 7 Pentium III 550 MHz
- 1.5 Gig RAM available for particle memory
- Particle decomposition networking
- Peak performance of 2 μs/particle/timestep (Cray C-90, one processor ~1.2)

Electrostatic Version

- Tridiagonal Poisson solver
- 2.5 dimensional (2 spatial, 3 velocity)
- Implicit electron gyromotion
- PIC code (precise particle tracking with gridded Phi field)
- Automatic load balancing

Electromagnetic Version

- Iterative Poisson-like magnetic field solver
- Added single component A_z and implicit inductive E_z
- Darwin approximation: no displacement current
- Only oblique propagation for electromagnetic waves (no A_x)

Gravitational Density Gradient

Two-Fluid Electrostatic Model

Momentum equation, both species

$$m\dot{\vec{v}} = -\vec{\nabla}P - qn\vec{\nabla}\Phi + mn\vec{g}$$

Poisson's equation

$$\frac{\partial^2\Phi}{\partial z^2} = -\frac{1}{\epsilon_0}(q_e n_e + q_i n_i)$$

Results

Equal exponential density profiles

$$n_e = n_o \, e^{-\gamma z}$$

$$n_i = n_o \, e^{-\gamma z}$$

Necessary Self-Consistent E field

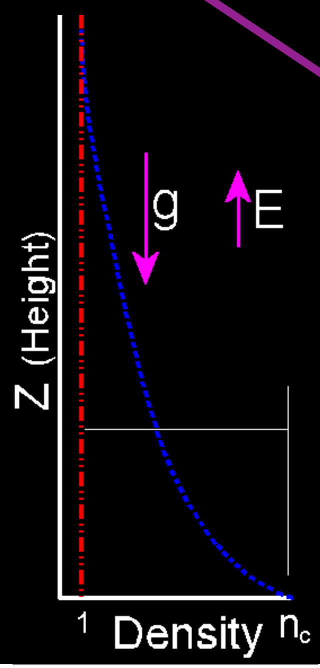
$$E_z = \frac{m_e g}{e}\frac{M - T}{T + 1}$$

$$\gamma \equiv \frac{m_e g}{T_e}\frac{M + 1}{T + 1}$$

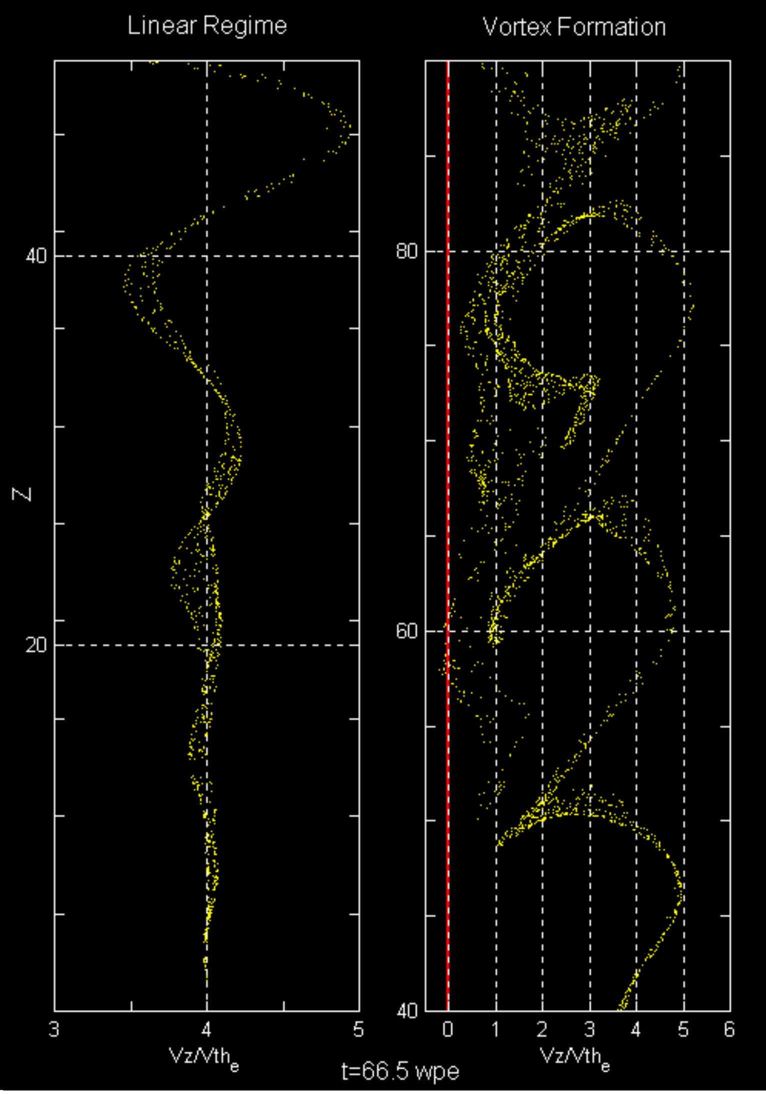
In real atmosphere, E ~ 1.2 μV/m

In simulation, ~ 1-2% of noise

$$T \equiv \frac{T_i}{T_e} \qquad M \equiv \frac{m_i}{m_e}$$



Electrostatic Beam Investigation: Solitary Waves

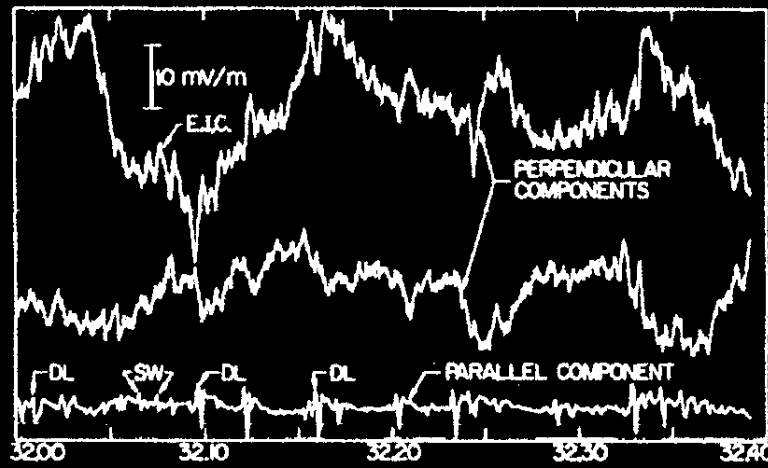


Cursory Solitary Wave History

• Name derives from curiously isolated nature not generally part of a periodic signal

• Temerin et al. (1982) first observed solitary waves in the upward current region via S3-3

• Later confirmed by: Viking in 88 in same region, Freja in 94 at a much lower altitude, and Polar in 97 at a height of 2 R_E



• Structures were bipolar in E, were negatively charged, traveled at 5-50 km/s, were between 5-50 λ_{De} in parallel extent, had amplitudes of 1-10 mV/m, and were interpreted as ion holes resulting from a variety of possible ion-scale instabilities.

• In the same data as above, Polar also noted briefer spikes (100 μs instead of 10 ms).

• Later confirmed by: Polar observations in 98 at 5-7 RE, in that same year FAST provided first high resolution examination

• Structures were again bipolar in E, were positively charged with a surrounding negative halo, traveled at 500-5000 km/s (5-50% v_{Te}), 10% density depletion with a Gaussian profile, were 1-10 λ_{De} in parallel extent, had amplitudes of 100 mV/m, and were interpreted as electron holes.

Solitary Wave Simulations

- Common simulation setup one: fully periodic, infinitely wide, pre-existing beam
- Used by Omura et al [94/96], Miyake et al [97/98/00], Goldman et al [99]
- Permits extremely long-time study of solitary structure formation, interaction, and decay
- Problems: constant particle number, no energy outflow, no global parallel structure, structures cannot escape from each other, only absolute instabilities possible

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- Common simulation setup two: injected, narrow beam
- Used by Singh [85/00]
- Permits convective instabilities to arise, short-time study of structures but long-time study of the region exposed to the incident beam
- Problems: Current simulations maintain too few particles (4 to 9 ppc) and temporal smoothing blurred solitary structure features. Later work moved to 3D instead of additional 2D resolution

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- In all cases, observed structures did form, interact, and decay
  Far too swiftly to explain space observations

Solitary Wave Simulations II

- Winglee et al. [88] attempted to introduce linear parallel density gradient

- Ignored equilibrium in bounded system, permitted fast electron beam to interact with slowly decaying density gradient

- Good first attempt at parallel gradients, but lack of equilibrium was highly detrimental to data interpretation

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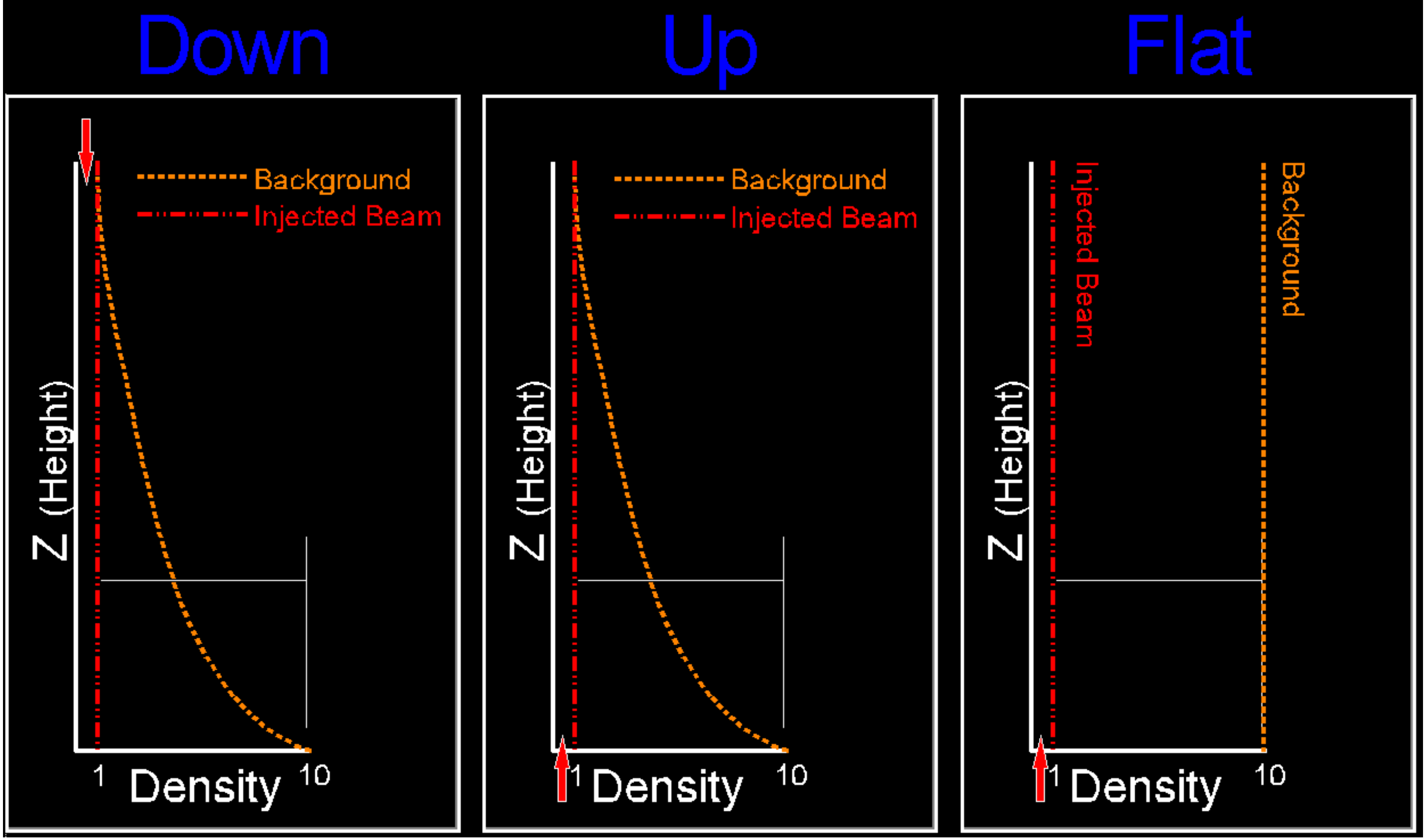
- Our dynamic particle number, parallel-bounded system can address the injected particle beam problem well

- Addition of a gravitational atmosphere permits equilibrium and hence a clean background against which beam interactions may be compared

- Specific areas of interest: special effects caused by parallel gradient and finite beam width

Beam Runs



Down
Up
Flat
Background
Injected Beam
Background
Injected Beam
Injected Beam
Background
Z (Height)
Z (Height)
Z (Height)
1
Density
10
1
Density
10
1
Density
10

Finite Beam Dispersion I

- Begin with standard cold, 3-fluid, electrostatic equations (electrons, ions, beam electrons)
  - 9 momentum equations
  - 3 continuity equations
  - Poisson's equation

- Utilize Fourier transform in z only – preserve x derivatives

- Solve for Phi:

Perpendicular magnetic terms

$$\frac{\partial}{\partial x}\left(\boxed{\left[1-\frac{\omega_{p_b}^2}{\bar{\omega}^2-\Omega_e^2}-\frac{\omega_{p_e}^2}{\omega^2-\Omega_e^2}-\frac{\omega_{p_i}^2}{\omega^2-\Omega_i^2}\right]}\frac{\partial\Phi}{\partial x}\right)$$

$$-K_{\|}^2\boxed{\left(1-\frac{\omega_{p_b}^2}{\bar{\omega}^2}-\frac{\omega_{p_e}^2}{\omega^2}-\frac{\omega_{p_i}^2}{\omega^2}\right)}\Phi=0$$

$$\bar{\omega}\equiv\omega-K_{\|}v_b$$

Standard 1D beam-plasma instability terms

Beam Model

- **Approximate narrow beam with infinitely thin width:** $\omega_{p_b}^2 \rightarrow \omega_{p_b}^2 \cdot Width \cdot \delta(x)$

- **Include parallel thermal effects:** $\omega_{p_e}^2 \rightarrow \omega_{p_e}^2 \left(1 + 3K_\parallel^2 \lambda_{D_e}^2\right)$

- **Neglect:** $\Omega_e^2 >> \bar{\omega}^2$ $\quad \dfrac{\omega_{p_b}^2}{\Omega_e^2} << 1 \ (0.04 \text{ in our setup})$

$$\frac{\partial}{\partial x}\left(\left[1 - \frac{\omega_{p_e}^2}{\omega^2 - \Omega_e^2} - \frac{\omega_{p_i}^2}{\omega^2 - \Omega_i^2}\right]\frac{\partial \Phi}{\partial x}\right) - K_\parallel^2\left(1 - \frac{\omega_{p_e}^2}{\omega^2} - \frac{\omega_{p_i}^2}{\omega^2}\right)\Phi$$
$$= -\frac{K_\parallel^2 \omega_{p_b}^2}{\bar{\omega}^2} \cdot Width \cdot \delta(x)\Phi$$

Finite Beam Dispersion II

- **Introduce Scaled Quantities:**

$$S \equiv \frac{v_b}{v_{T_e}},\ L \equiv \frac{Width \cdot \omega_{p_b}}{v_b},\ N \equiv \frac{\omega_{p_b}^2}{\omega_{p_e}^2} = \frac{n_b}{n_e},\ d \equiv \frac{m}{M},\ \alpha \equiv \frac{\Omega_e}{\omega_{p_e}},\ W \equiv \frac{\omega}{\omega_{p_e}},\ K \equiv \frac{K_\parallel v_b}{\omega_{p_e}}$$

- **Assume decaying exponential solution outside beam:**

$$\text{For } x >< 0, \text{ let } \Phi_{><} = \Phi_0 e^{\mp K_\perp x}$$

- **Plug in outside region to obtain perp/parallel K relation:**

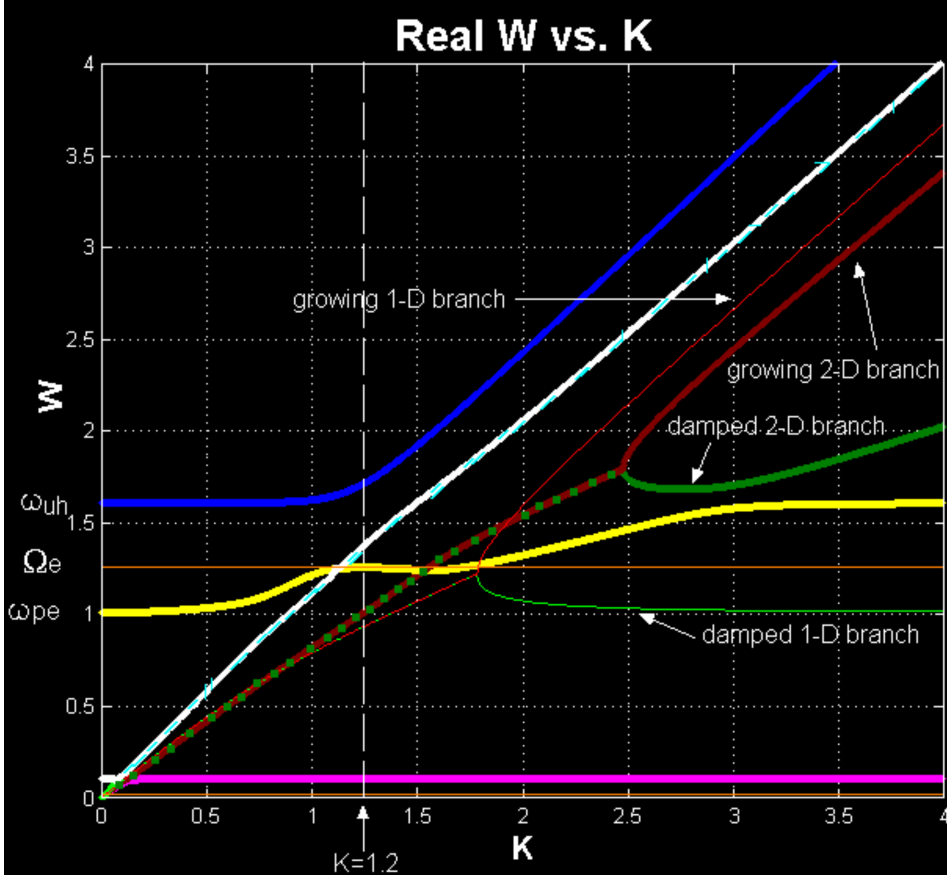
$$K_\perp = K_\parallel \sqrt{\frac{W^2 - (1+d)(1+3K^2)}{W^2\left(1 + \frac{(1+d)(d\alpha^2 - W^2)}{(W^2-\alpha^2)(W^2-d^2\alpha^2)}\right)}}$$

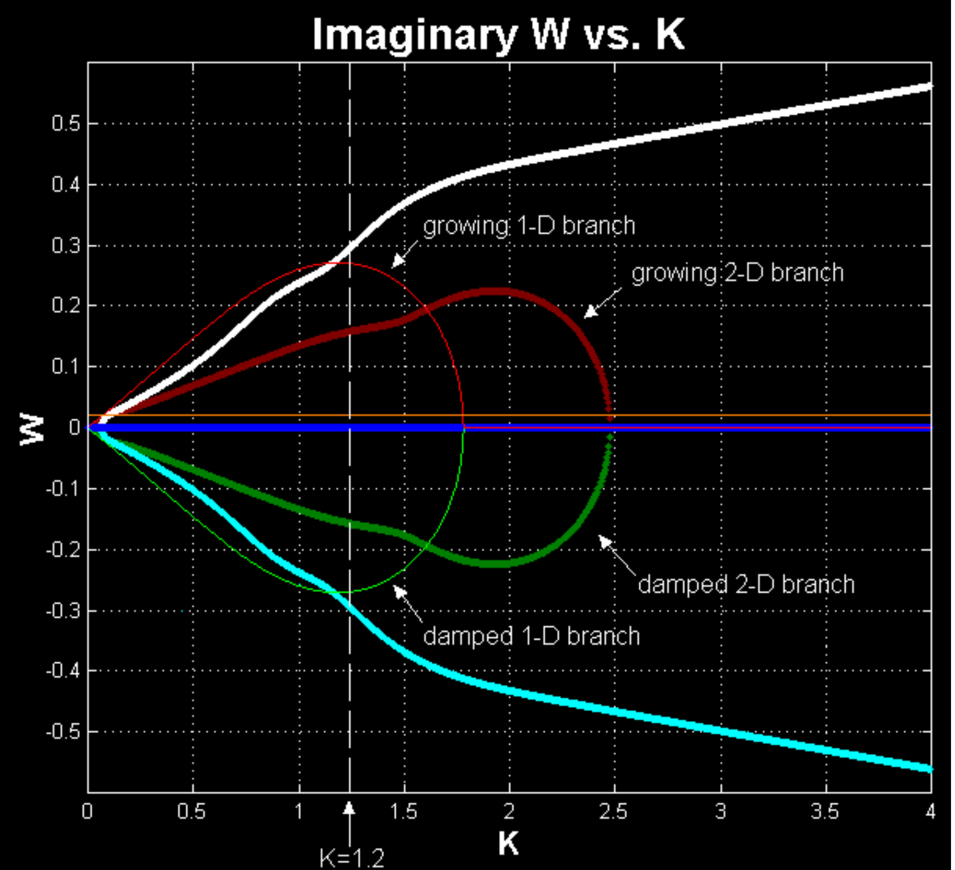
- **Integrate resulting equation across delta function to obtain final dispersion relation:**

$$4\left[-(1+d)(3K^2+S^2) + S^2W^2\right]\left[1 + \frac{(1+d)(d\alpha^2 - W^2)}{(W^2-\alpha^2)(W^2-d^2\alpha^2)}\right] = S^2L^2N\frac{W^2K^2}{(K-W)^4}$$

Finite Beam Dispersion Graph

- **1-D vs. Narrow Beam Dispersion Graphs: Complex W vs. Real K**

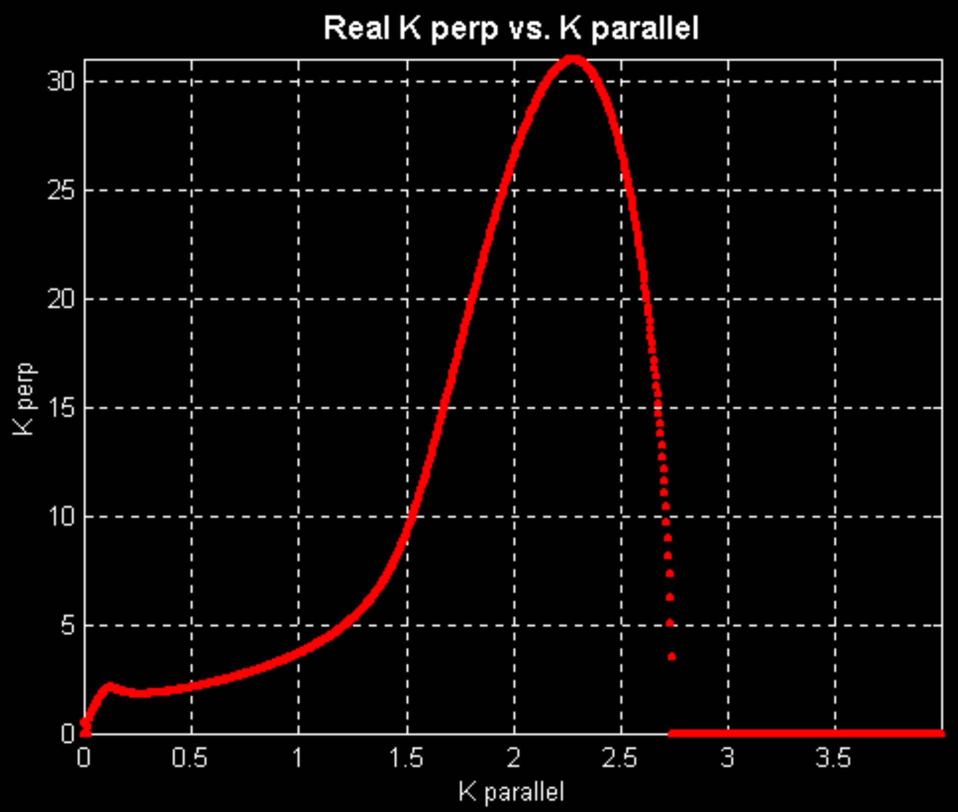




- **Many new modes, 2 of which are growing**
- **White/cyan supra-beam mode has negative (damped) real K - doesn't match B.C.'s**
- **Model predicts smaller wavelengths are favored by finite beams**

Finite Beam K Damping

- Graph of Growing Beam Mode
- Recall real, positive K perp is a damping term



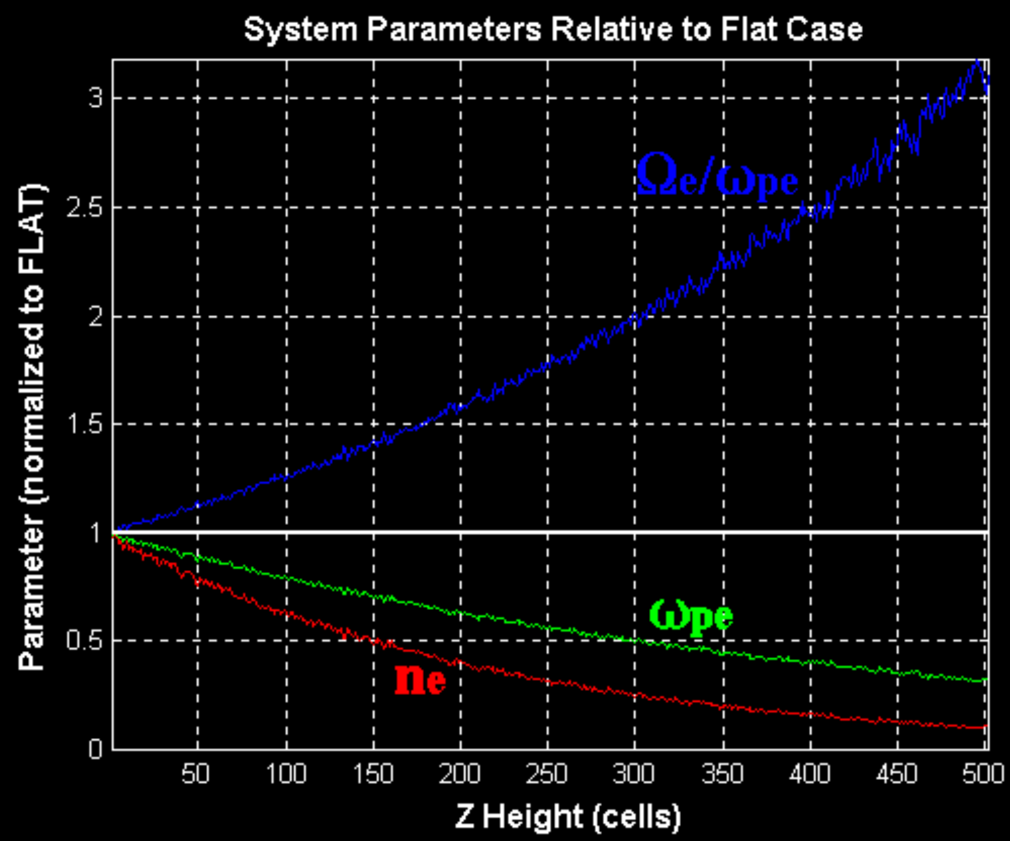
- Small wavelengths are rapidly damped out
- Longer wavelengths extend deeper into background

Beam Simulation Parameters

Flat Case

- Δ = 1.4 λ_{De}
- β = 0.0013
- Ω_e/ω_{pe} = 1.25
- Ω_i/ω_{pi} = 0.16
- Ω_e/ω_{pe} = 5.00
- m/M = 0.016 (1/64)
- T_i/T_e = 1
- T_b/T_e = 0.002 (1/500)
- n_b/n_e = 0.1 (1/10)
- $\Delta t \cdot \omega_{pe}$ = 0.13
- Width = 6Δ = 8.4 λ_{De}

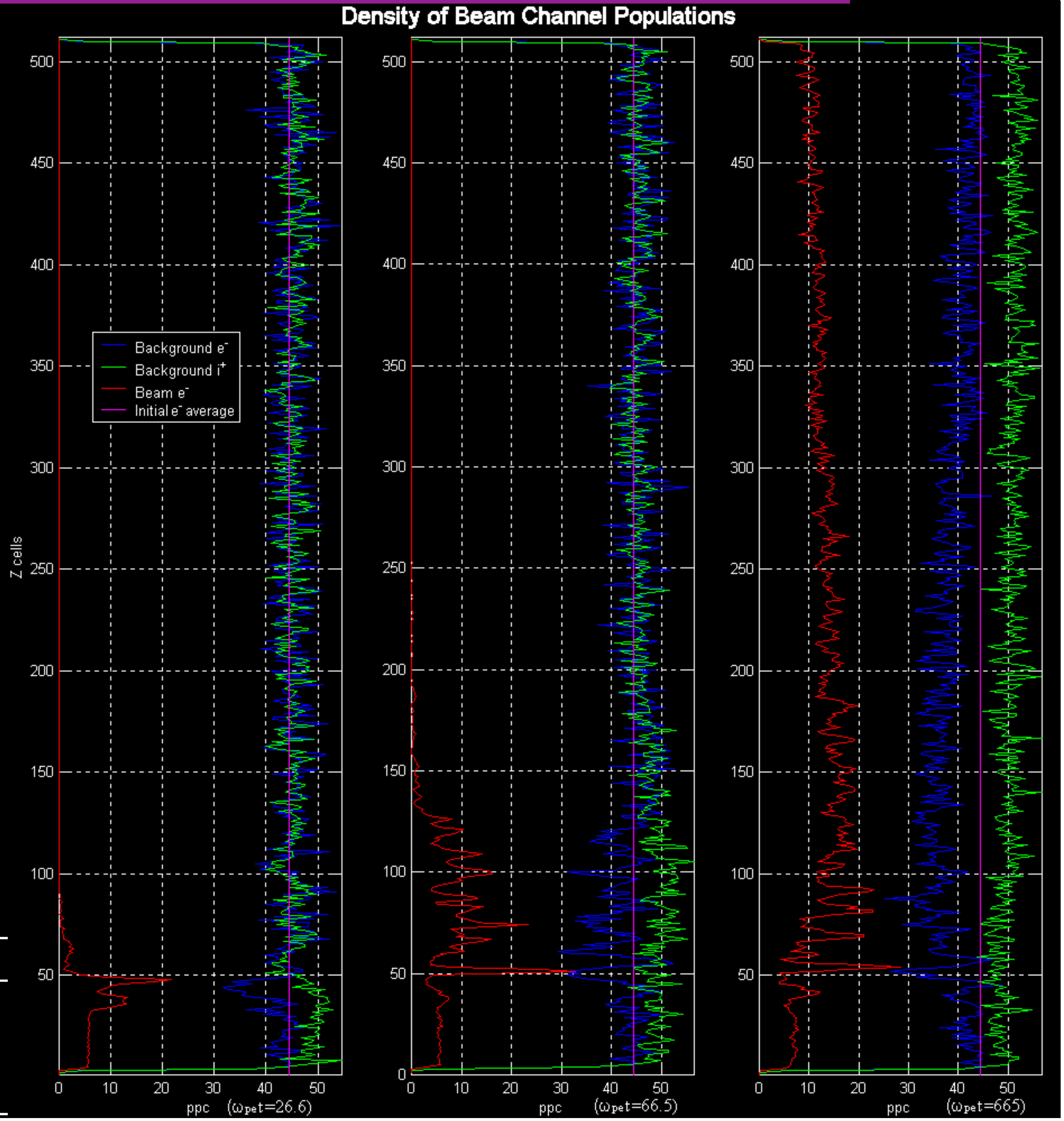
Gradient Cases



Beam Case 1/3:

The Flat Run

Flat Population Density Modification

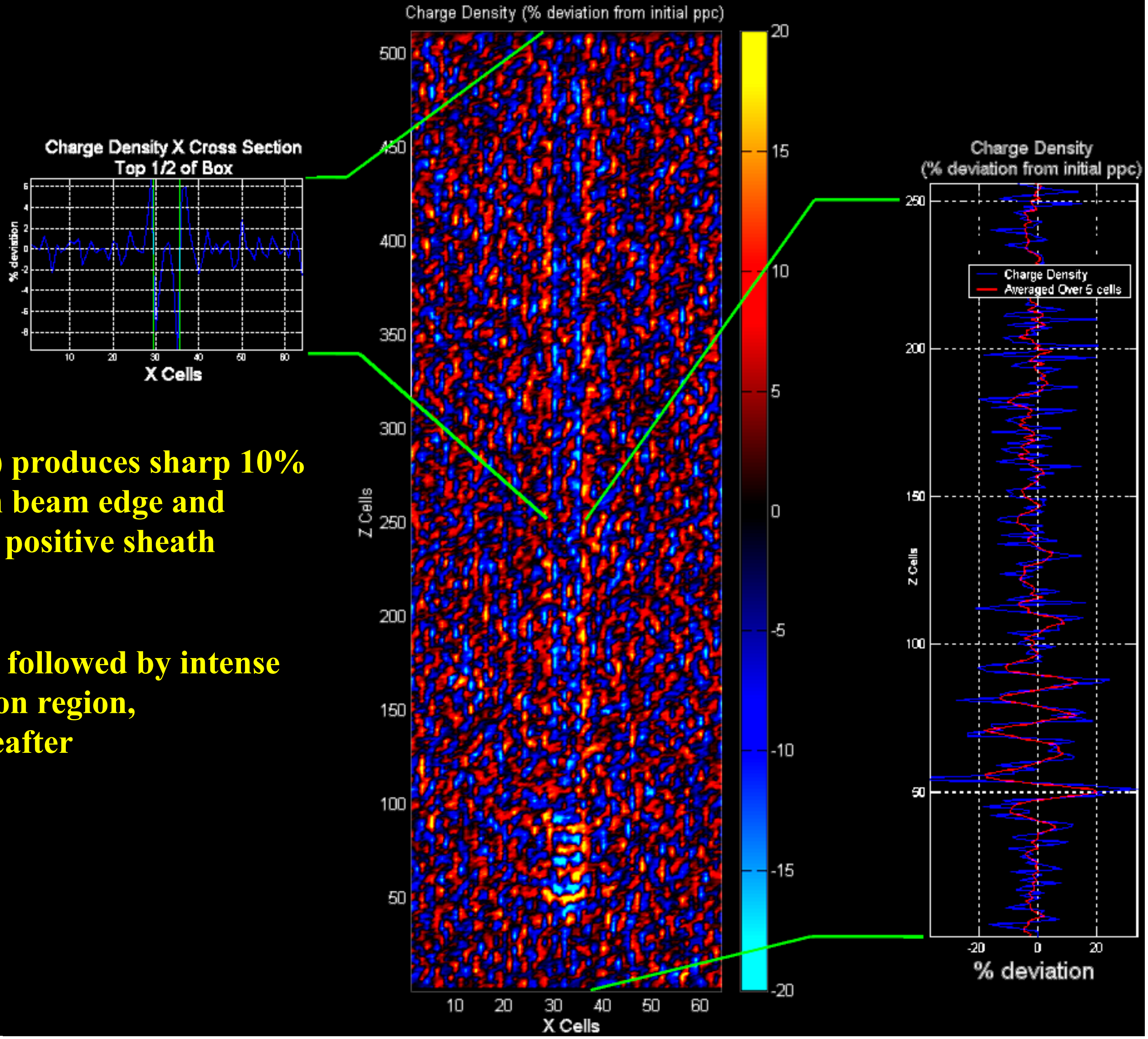


- **Graph of PPC vs. Height**
- **Injected beam charge is neutralized by both background species**
- **Intense interaction region followed by quiet, dense, warm beam**

Population	β	$1/\Omega_c$	$1/\omega_p$	v_d	r_{gyro}	λ_D
e- (back)	5e-4	6.0	12.0	0	0.57	1.14
i+ (back)	5e-4	384	96.0	0	4.50	1.14
e- (beam)	1.6e-7	6.0	29.9	4	.025	0.13

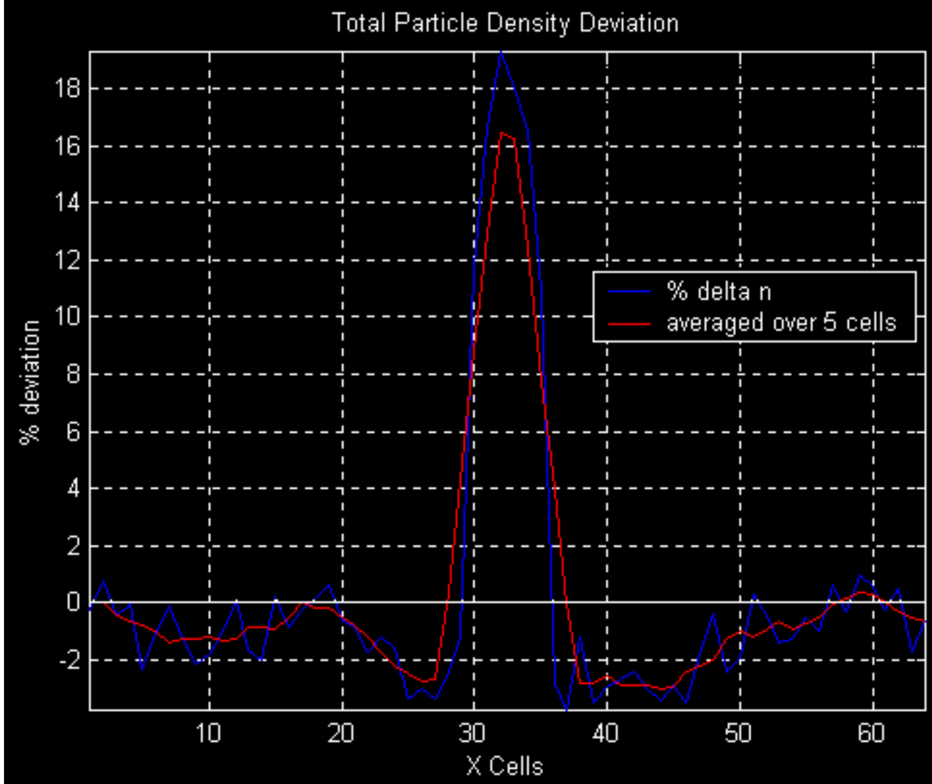
Charge Density Modification

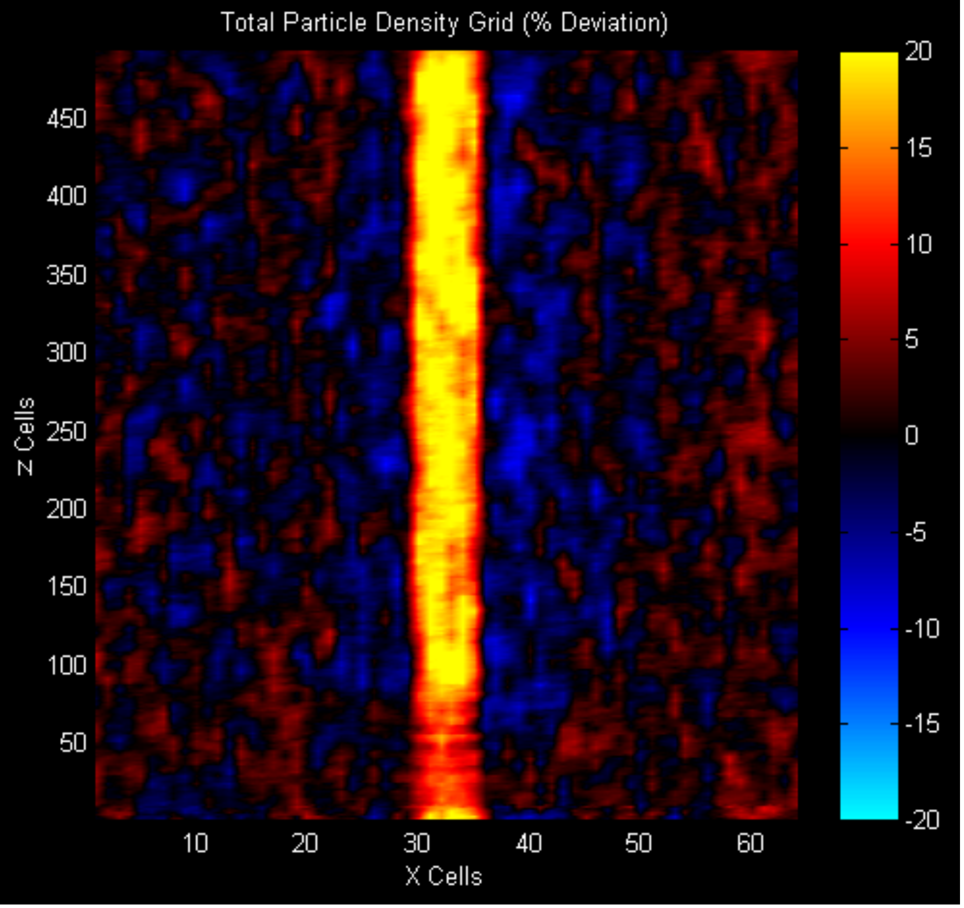
- Beam is overall neutralized within 0.3%
- ρ_i vs. ρ_e (4.5Δ vs. 0.6Δ) produces sharp 10% negative sheath within beam edge and wider, less intense 6% positive sheath outside beam
- Quiet injection region followed by intense electrostatic interaction region, then quiet beam thereafter



Flat Total Density Modification

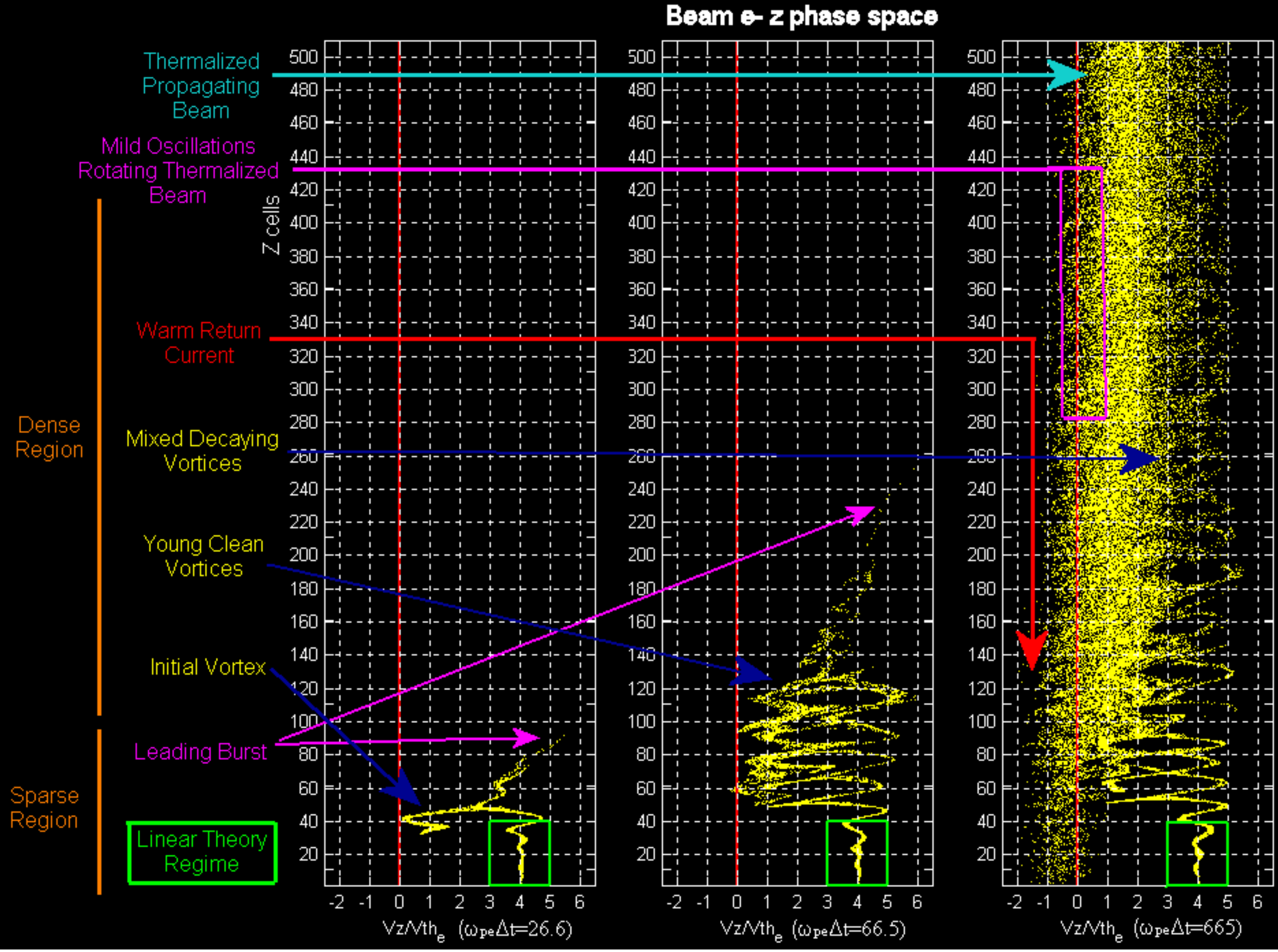
- Injection of beam electrons increases total plasma density along beam channel up to 20%
- Plasma density depletion sheath forms around beam for 10's of λ_{De} with max 2%





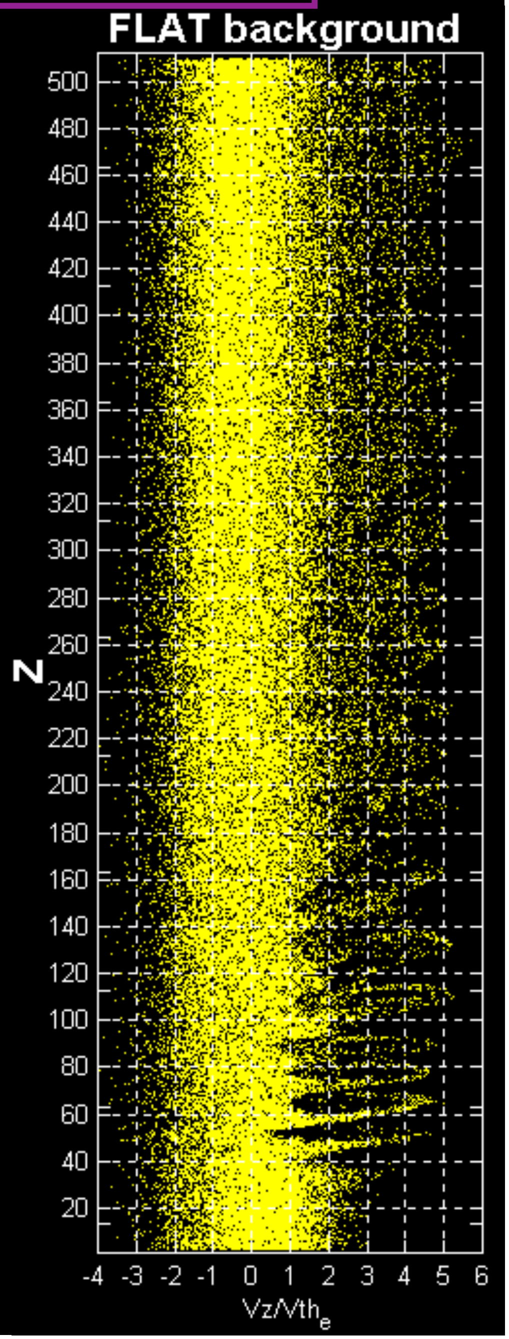
Movie

Flat Z Phase Space



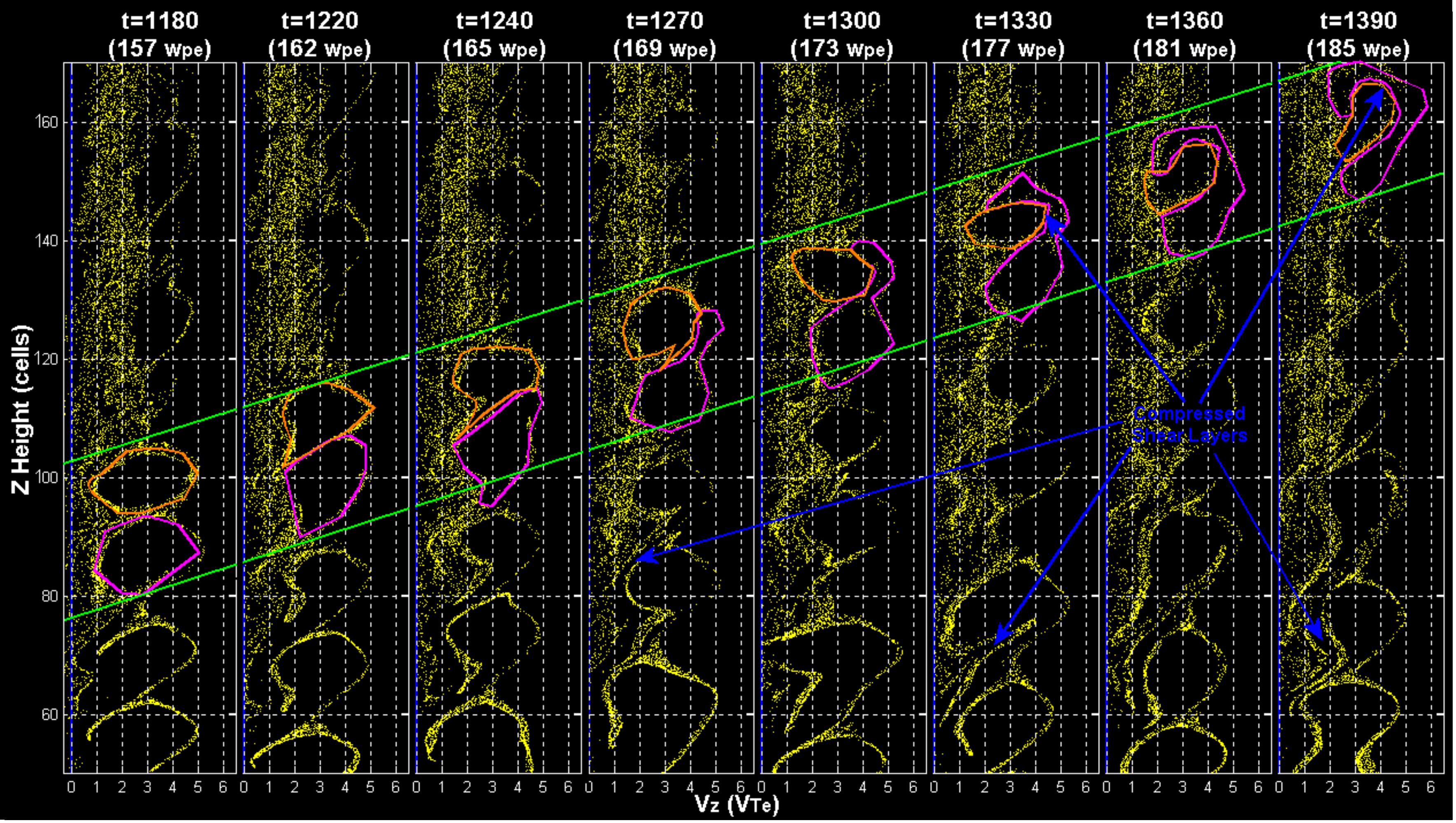
Flat Z Phase Space II

- Leading burst
  - -small population (<1% of beam population)
  - -decelerates from 4 v_{Te} to 2.7
  - -accelerates up to 6 v_{Te}
  - -transient effect only: quickly gone and would soon leave beam region in space
- Vortices form but rapidly decay
- Warm return current is generated as vortices decay
- Perpendicular temperature is warmed to around 1/9 T_e
- Beam is completely thermalized ½ way through box
- Background population snapshot shows heavy participation
  Background particles are trapped; beam particles are not
- Empirical results: W=1.02 ± 0.01 K=1.2 ± 0.3
- Linear theory for W(K)=1.0 ± 0.2, good match



Vortex Decay Detail

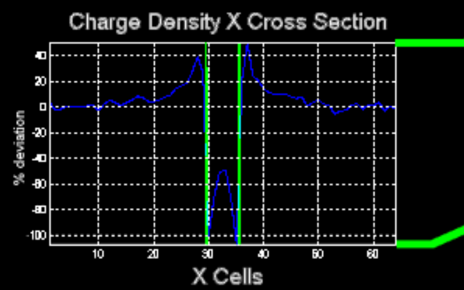
- Vortex Mergers
  - -Similar speed and size of vortices causes mergers to be destructive
- Destruction of Free Energy Source
  - -Beam is thermalized too rapidly and cannot penetrate the box to sustain vortices



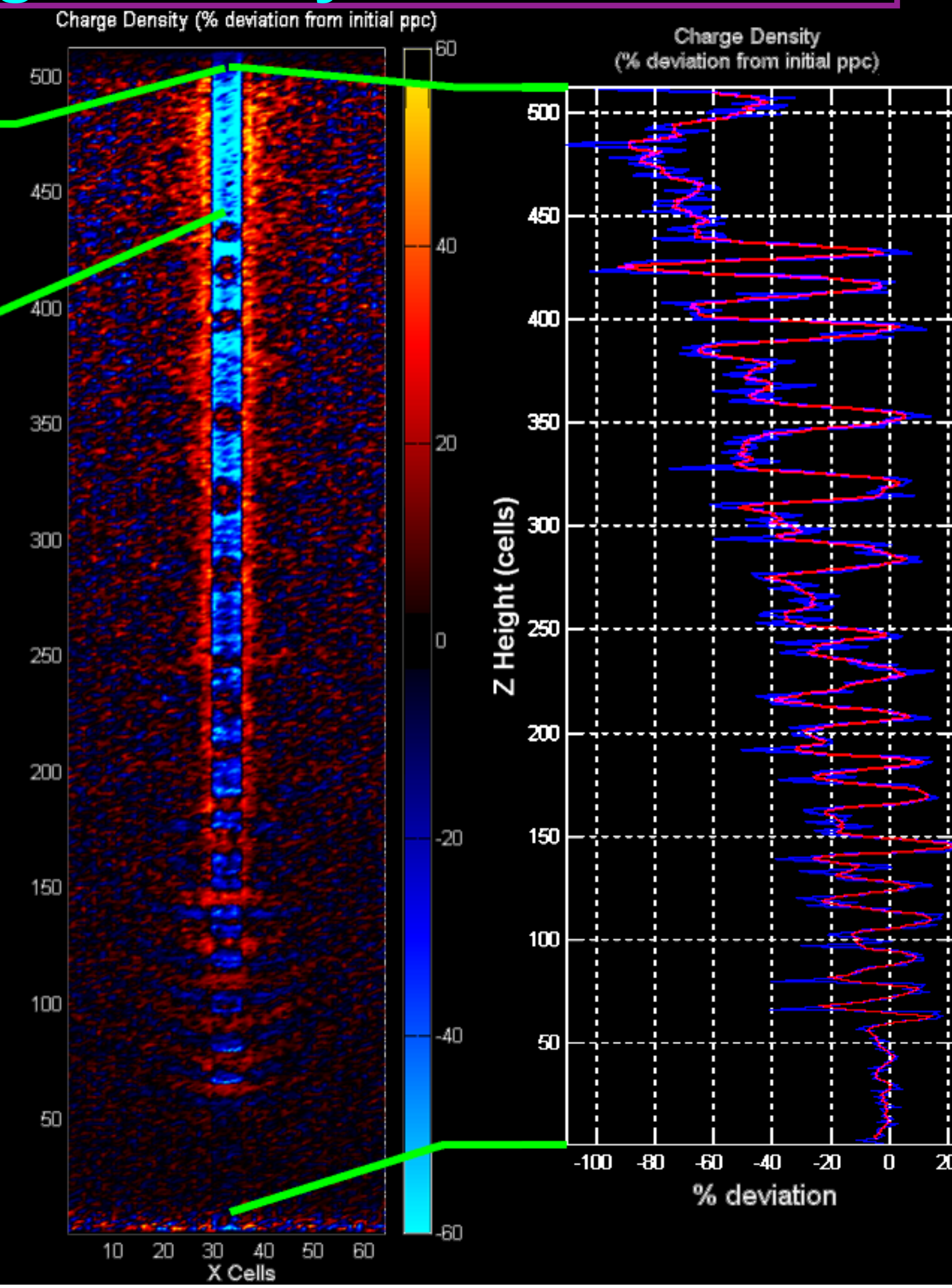
Beam Case 2/3:

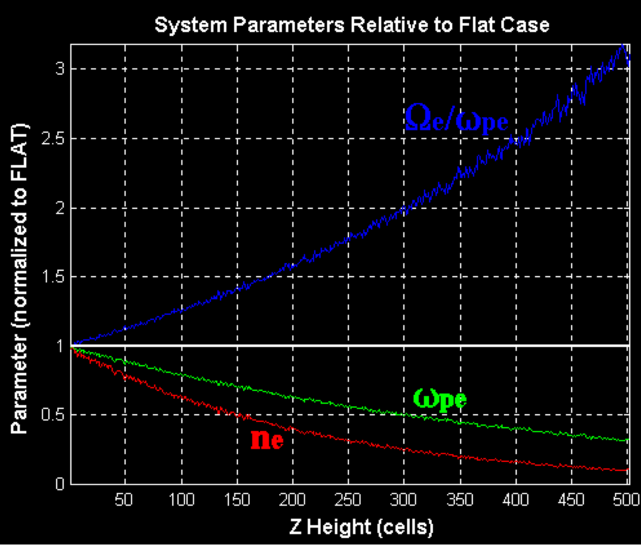
The Up Run

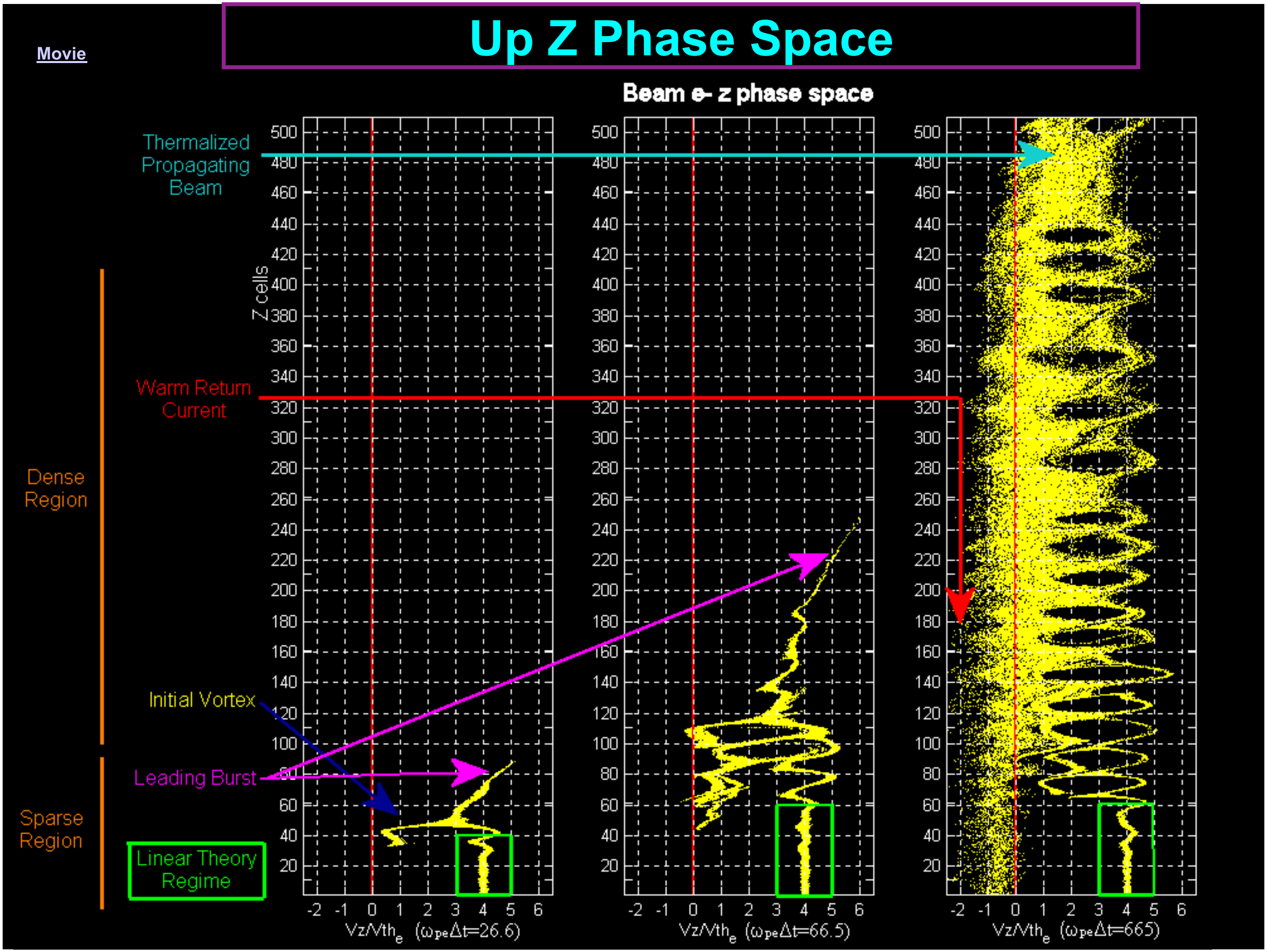
Up Population Charge Density Modification



- Beam easily clears out density channel
- Sparse magnetosphere unable to neutralize beam as efficiently
- Structures persist throughout box



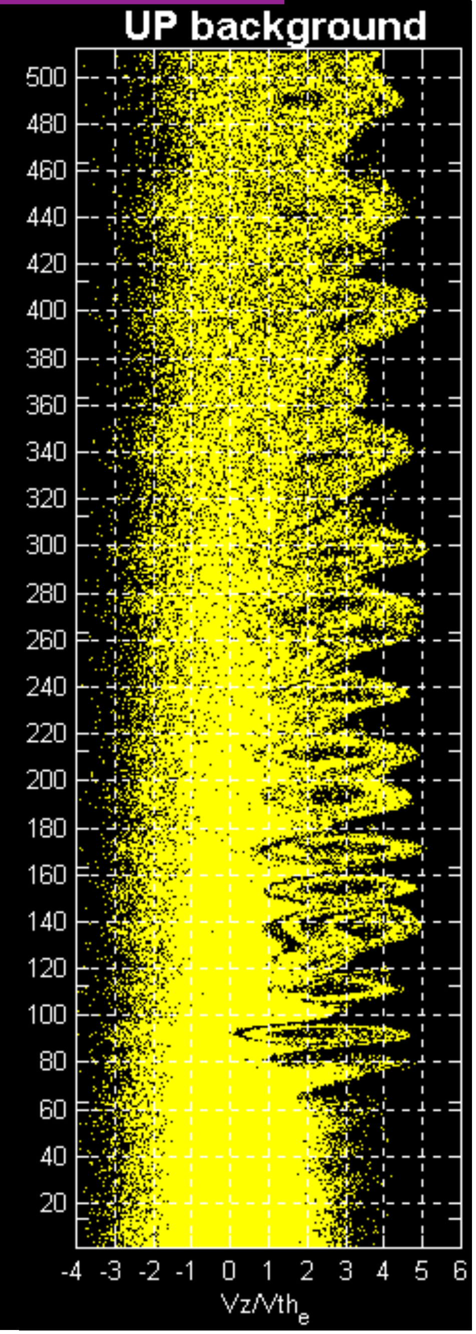




Movie
Up Z Phase Space
Beam e- z phase space
Thermalized Propagating Beam
Warm Return Current
Dense Region
Initial Vortex
Leading Burst
Sparse Region
Linear Theory Regime
Z cells
Vz/Vth$_e$ ($\omega_{pe}\Delta t$=26.6)
Vz/Vth$_e$ ($\omega_{pe}\Delta t$=66.5)
Vz/Vth$_e$ ($\omega_{pe}\Delta t$=665)

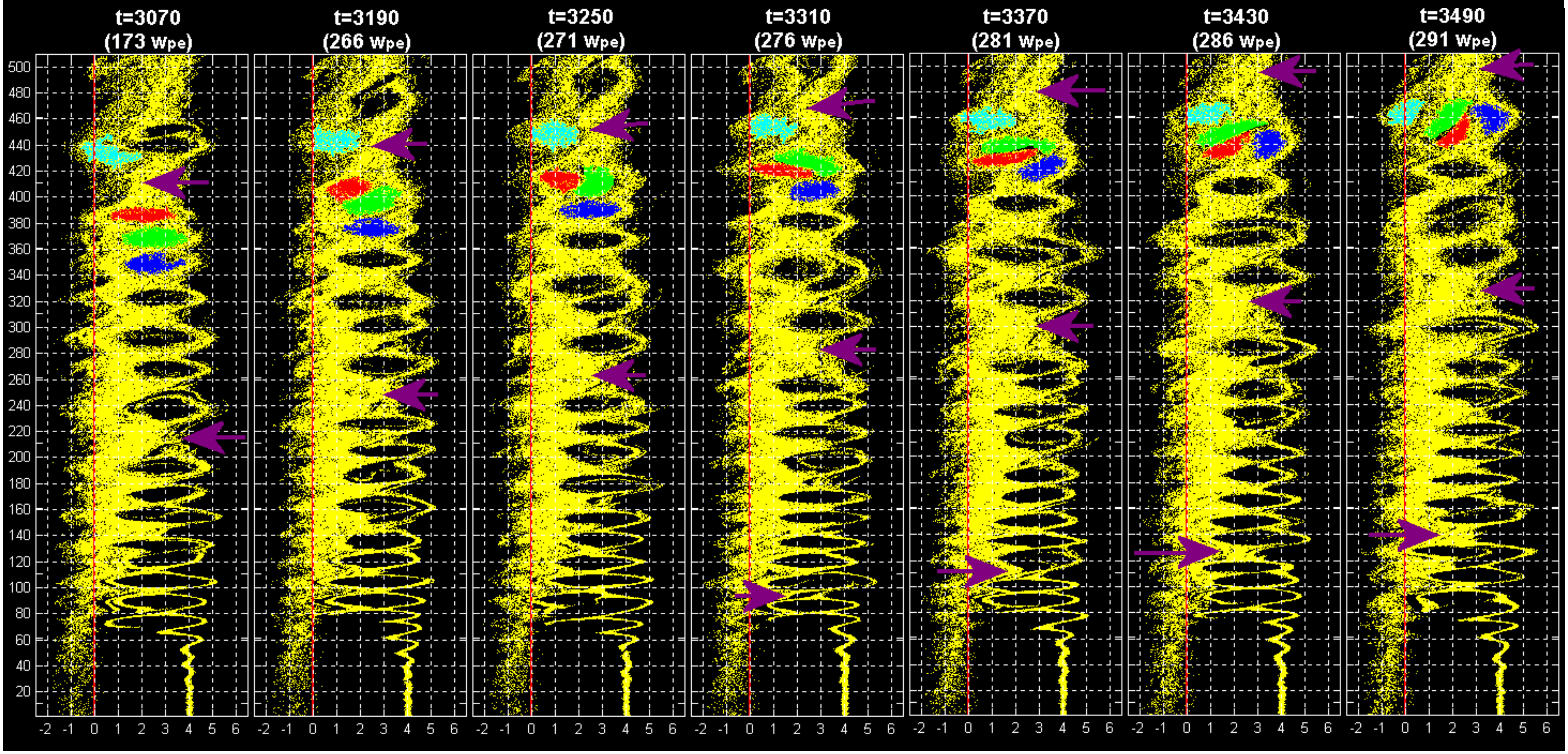
Up Z Phase Space II

- Vortices form and remain remarkably coherent
- Enhanced warm return current
- Significant fraction of the beam is NOT thermalized, but remains coherent along perimeter of vortices
- Background serves as trapped, phase-mixed population supporting BGK mode



Up Vortex Mergers

- Vortices clump together into complex structures without losing coherence
- Clumping rarefies structures along beam, producing gaps and randomizing distribution
- Possible mechanism for conversion of periodic signal to space-like solitary structures
- Some vortices actually move counter-stream for many ion gyroperiods
- Beam complexity provides variation in size and hence speed of vortices
- Downward acceleration presses vortices together and enhances interaction frequency

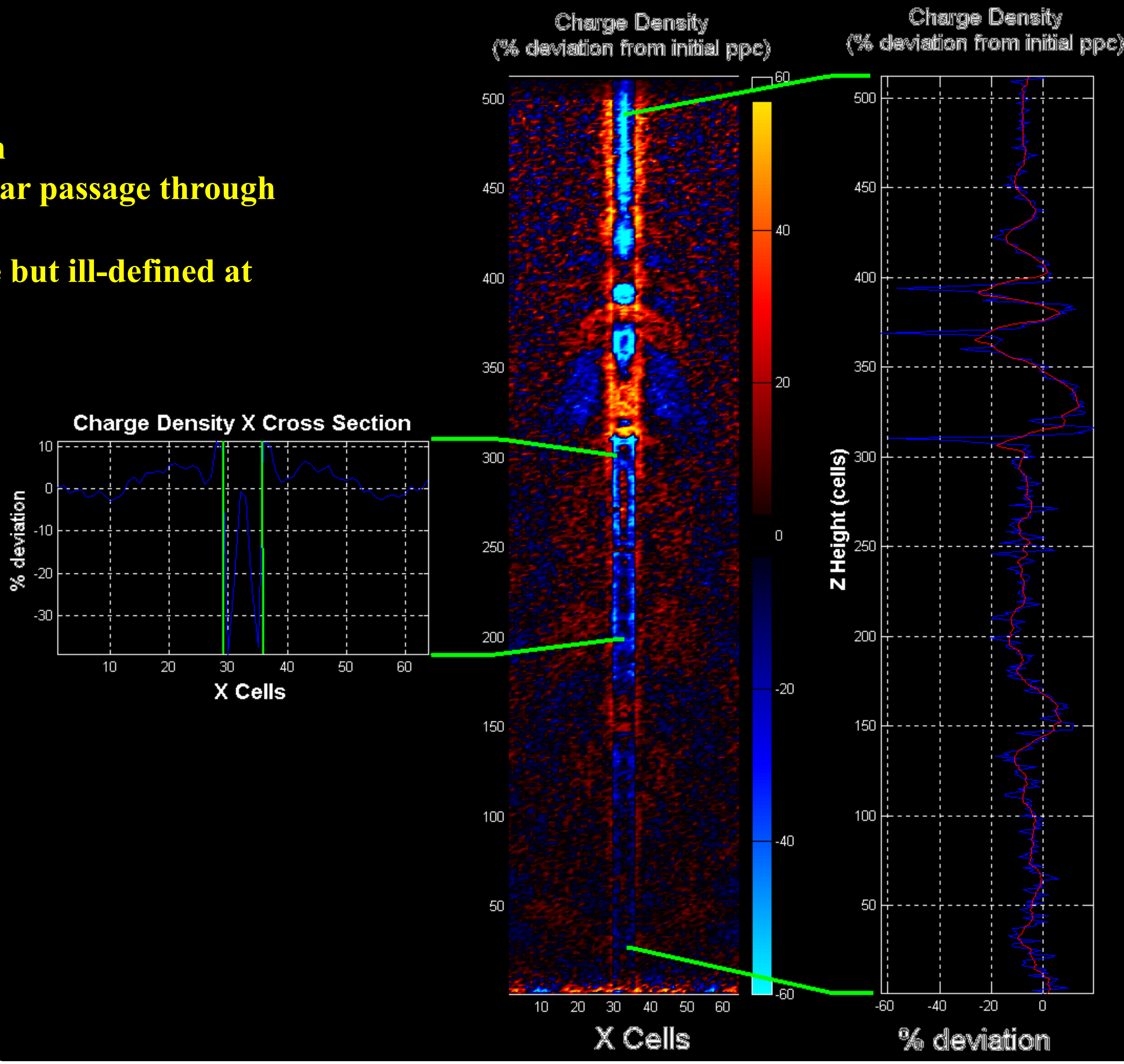


Beam Case 3/3:

The Down Run

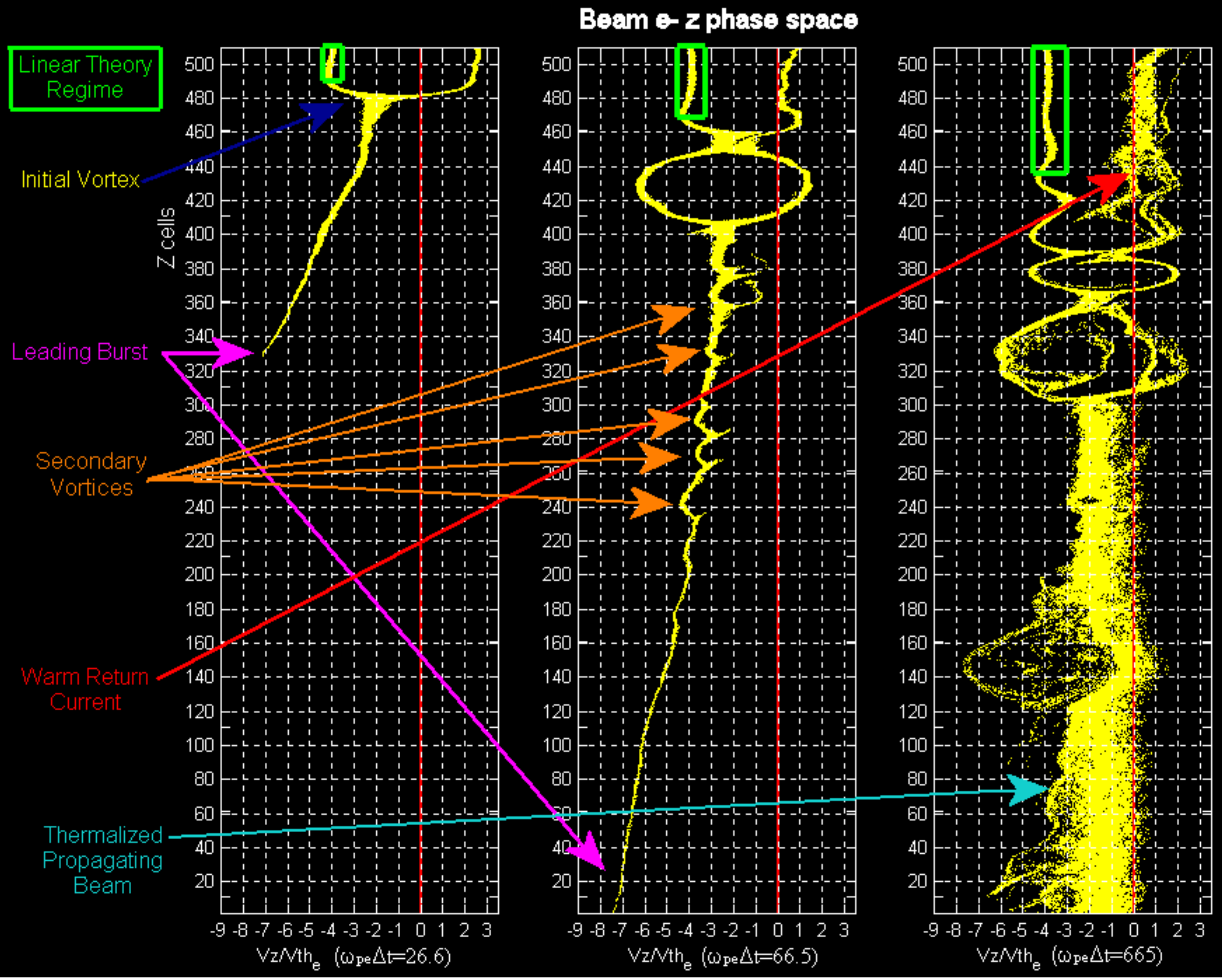
Down Population Charge Density Modification

- Huge structures form
- Beam is unable to clear passage through dense ionosphere
- Structures are sparse but ill-defined at low altitude



Movie

Down Z Phase Space



Down Z Phase Space II

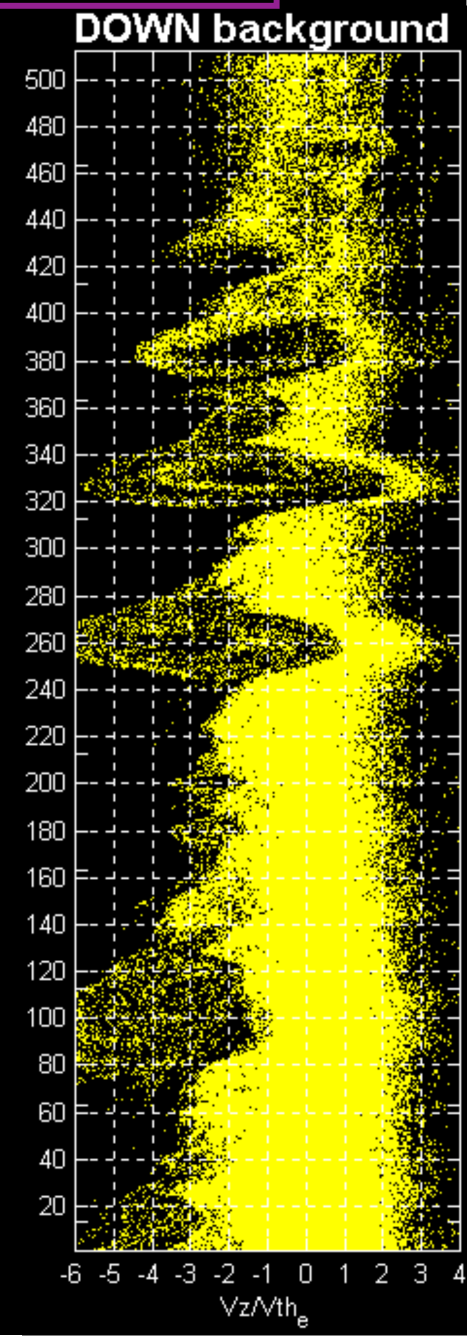
• Vortices are enormous and remain somewhat coherent through box

• Extremely strong, cool return current

• Large vortices disintegrate rapidly as they proceed into the gradient

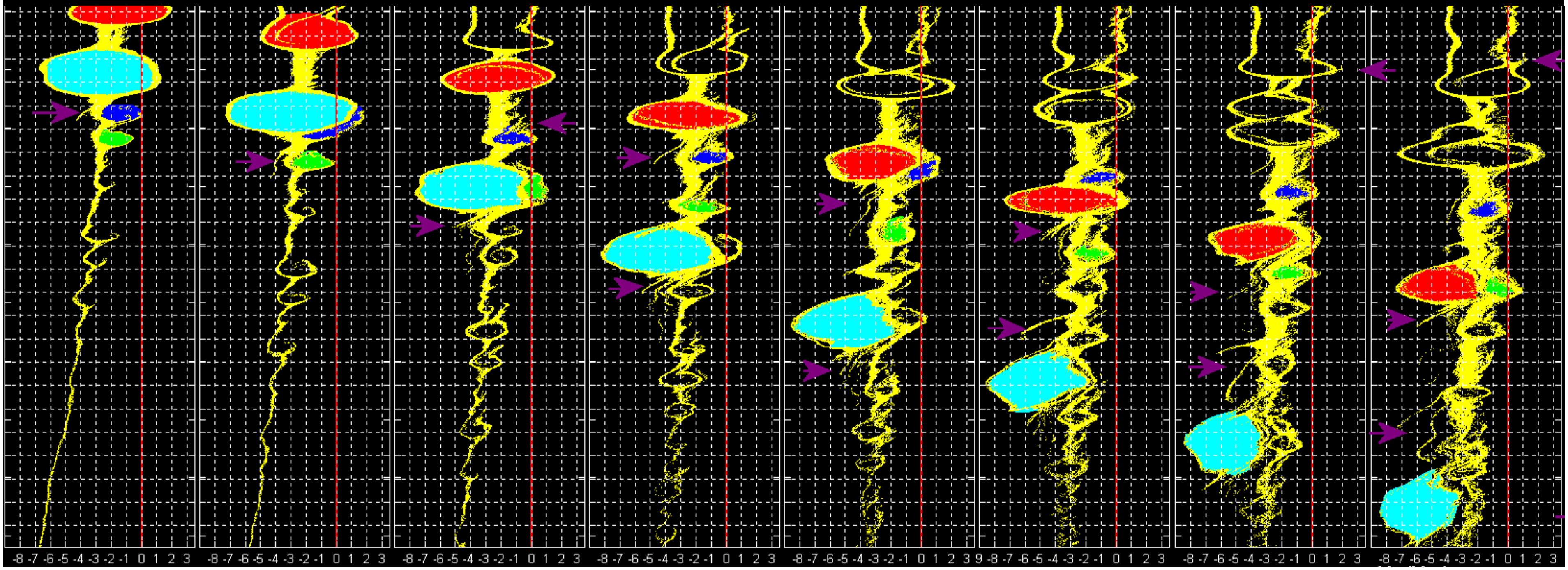
• Smaller, absolute vortices form are swept over by larger vortices

• Background is highly modified by enormous BGK support fields



Down Vortex Mergers

- Large vortices space out and rarely interact
- Smaller, separately formed vortices are simply swept past larger
- Fingers of accelerated particles reach out from interacting vortices
- Some secondary beams strong enough to produce microvortices
- Vortex destabilization due to lack of beam support after downward acceleration



Beam Comparisons

Solitary Wave
Conclusions

Effects Due to Finite Beam

- Robust BGK vortices form easily in beams of extremely narrow width
- Behavior is not fundamentally different than 1D instabilities
- No new physical results were interpreted as resulting from the finite nature of the beam

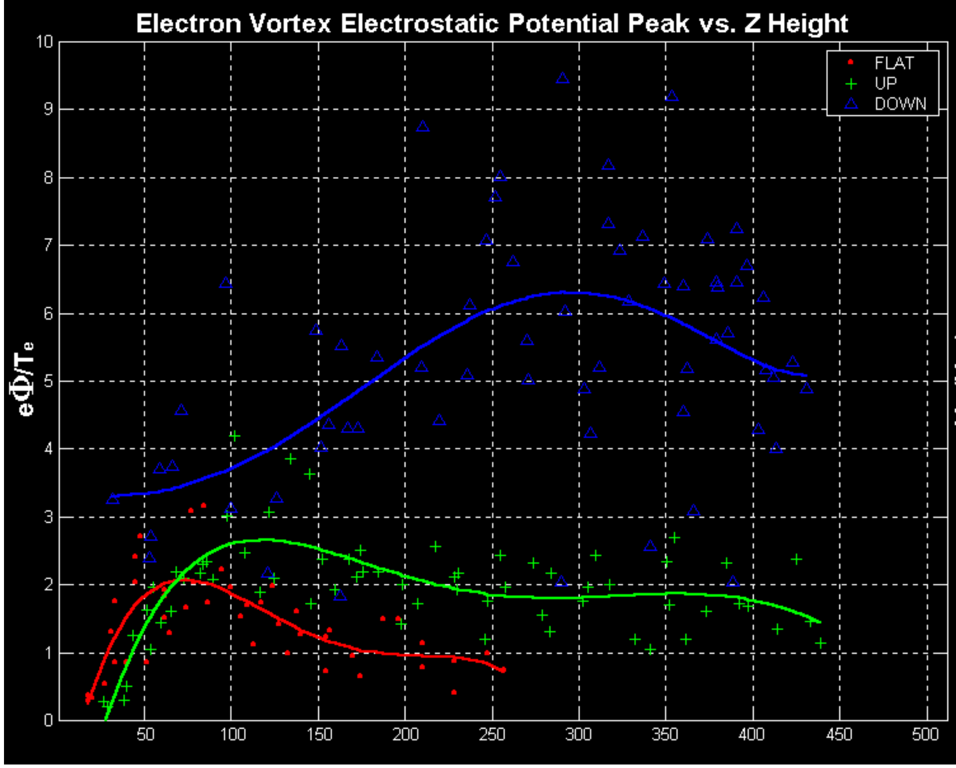
Effects Due to Convective Beam

- Beam particles remain untrapped while background phase mix
- Permits beam to remain cool even while participating in structures

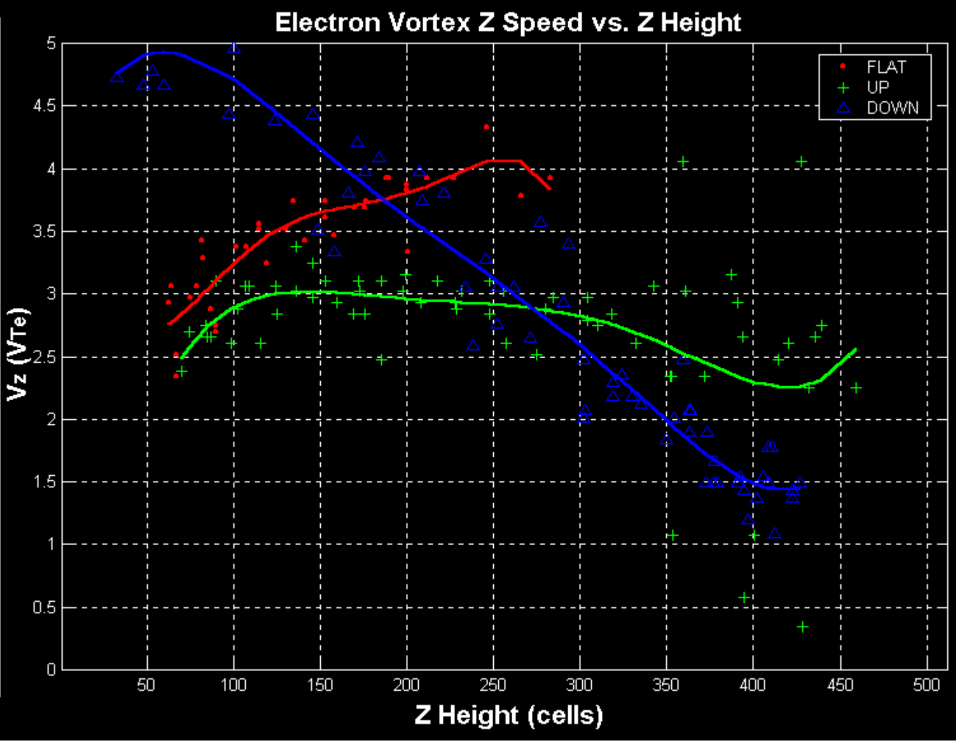
Effects Due to Parallel Density Gradient I

- Vortex formation occurs in presence of gradient
- Moving out of gradient causes vortex rarefaction by clumping
- Moving into gradient causes vortex rarefaction by direct acceleration

- Amplitude always decays

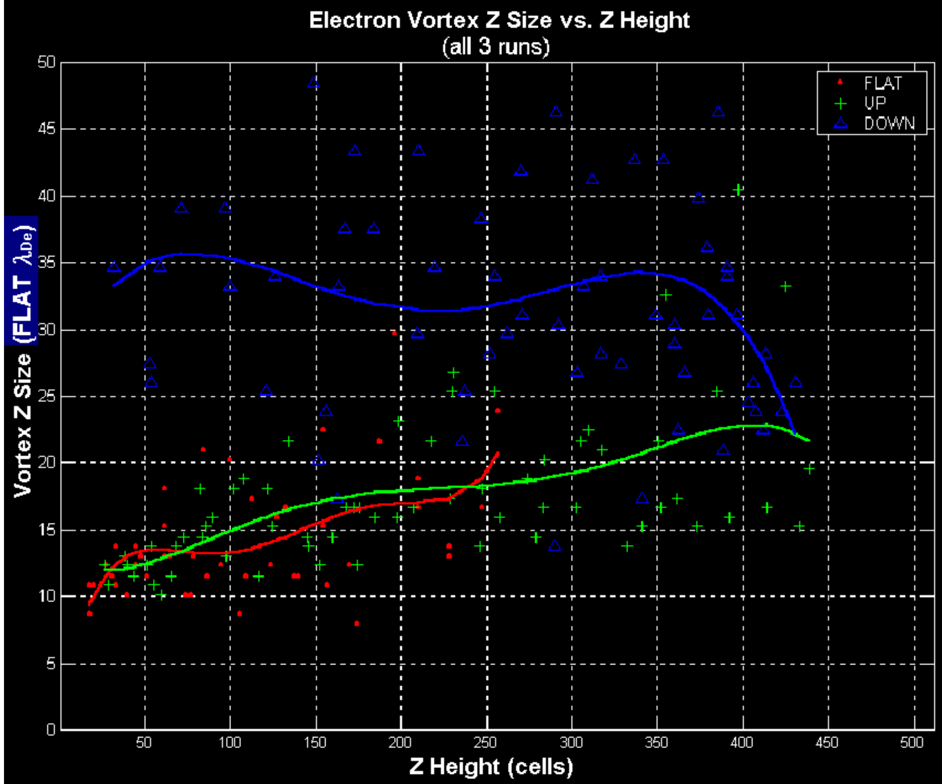


- Gradient-ward acceleration
  - Flat case is itself passing into self-made 1.5 gradient

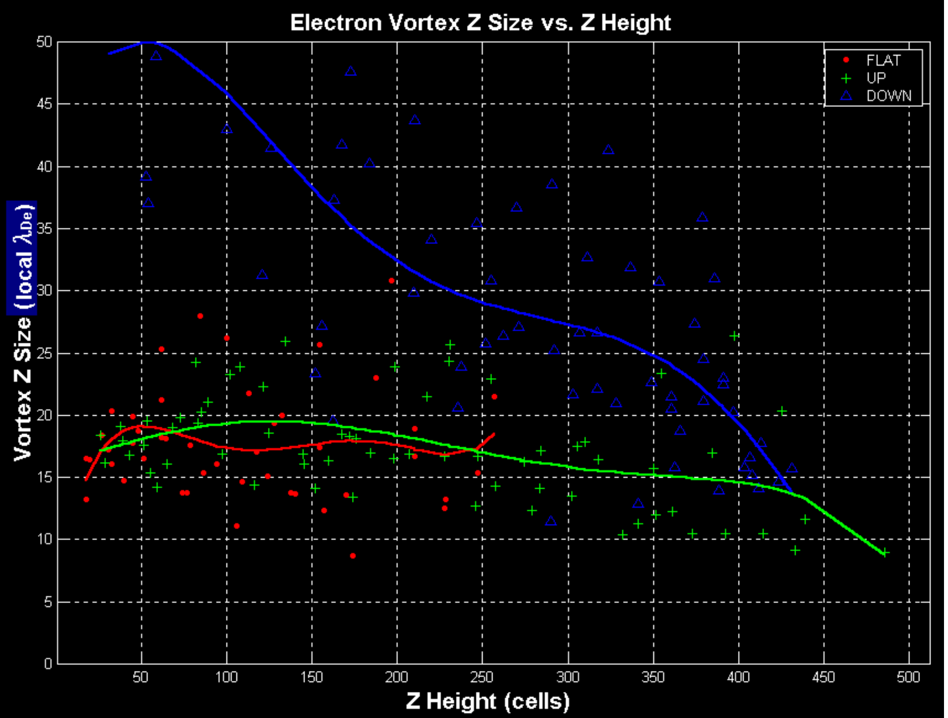


Effects Due to Parallel Density Gradient II

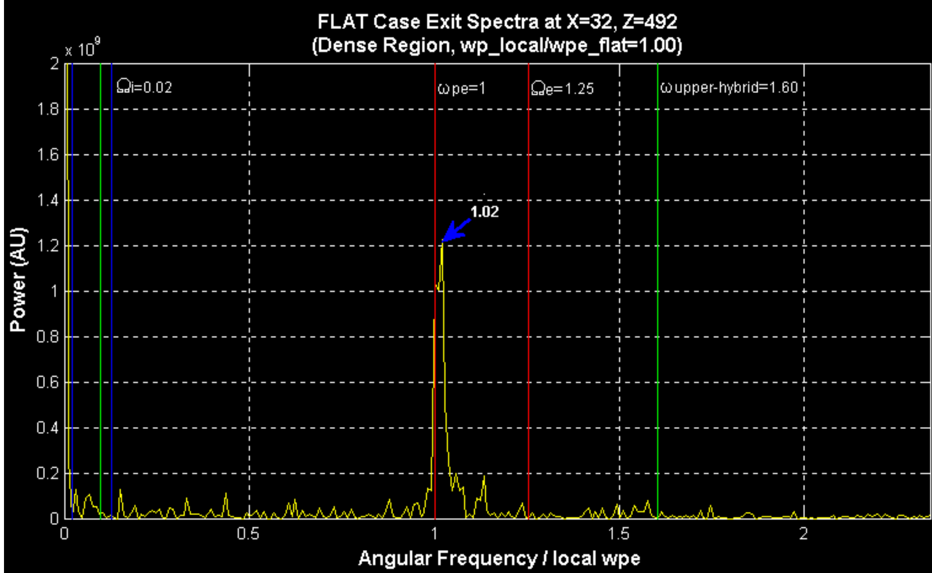
- Size always increases



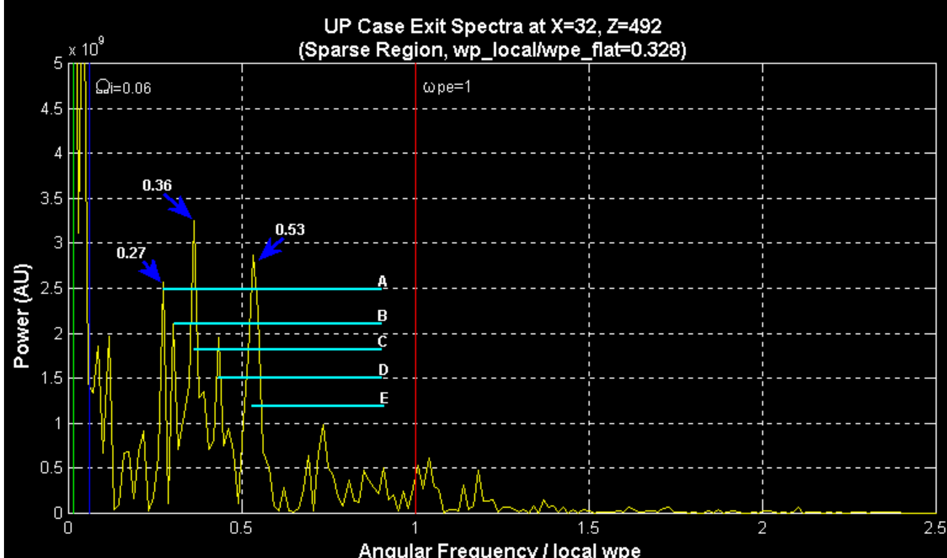
- Not when scaled to local λ_{De}

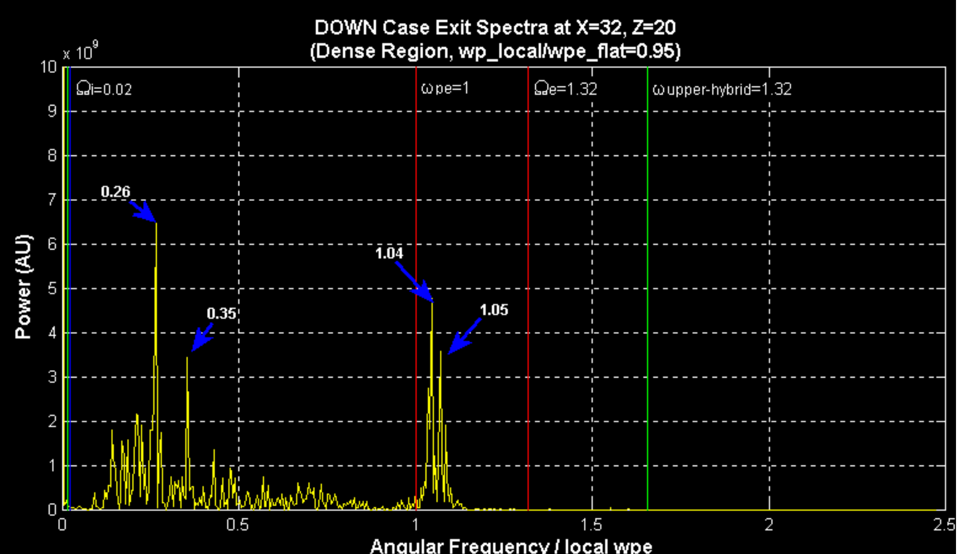


Exit Spectrum



- **High-density noise peak at ω_{pe} in FLAT/DOWN**
- **DOWN also shows large vortex exit frequency (0.26) and bounce frequency (0.35)**
- **UP run shows complex mixture of peaks corresponding to different sized combined structures**





Oblique Inertial Alfvén Wave Study

Cursory Auroral Alfvén Wave History

- At low altitudes, rocket flights observe Alfvén flux of ~0.1 to 10 ergs/s/cm2 and little reflection
  [Gurnett et al. 1984] [Boehm et al. 1990]

- At high altitudes (4-6 RE), Polar finds downward Alfvénic flux that would map to 100 ergs/s/cm2
  [Wygant et al. 1996]

- Waves appear to have enough energy to support the entire auroral system
  [Wygant et al. 1996]

- Freja noted small transverse scales on the order of the electron inertial length (~1 km)
  [Louarn et al.. 1994]

- Excellent candidate for energy transport from tail region to auroral region

- Oblique Alfven waves carry intrinsic parallel electric field
  might be source of parallel potential

- Since little reflection, must be absorbed by collisionless plasma

Standard Inertial Oblique Alfvénic Dispersion

- **Begin with standard cold, 2-fluid, electromagnetic equations**
  - **6 momentum equations**
  - **2 continuity equations**
  - **Ampere's Law**
  - **Faraday's Law**

- **Arrive at 10th order equation**

- **Taking limits**

  W<<1 (low frequency)

  d<<1 (large ion mass)

  $K_{\parallel} << K_{\perp}$ (oblique propagation)

$$r \equiv \frac{v_a}{c} \qquad K \equiv \frac{c}{\omega_{pe}} k \qquad W = \frac{\omega}{\omega_{pe}}$$

- **Standard inertial Alfvén relation:**

$$W^2 = \frac{K_{\parallel}^2 r^2}{1 + K_{\perp}^2}$$

Simulation Inertial Oblique Alfvénic Dispersion

- Begin with standard cold, 2-fluid, Darwin-approximated electromagnetic equations
  - 6 momentum equations
  - 2 continuity equations
  - Poisson's equation
  - Ampere's Law w/o displacement current (A_z component only)
  - Faraday's Law (E_z component only)

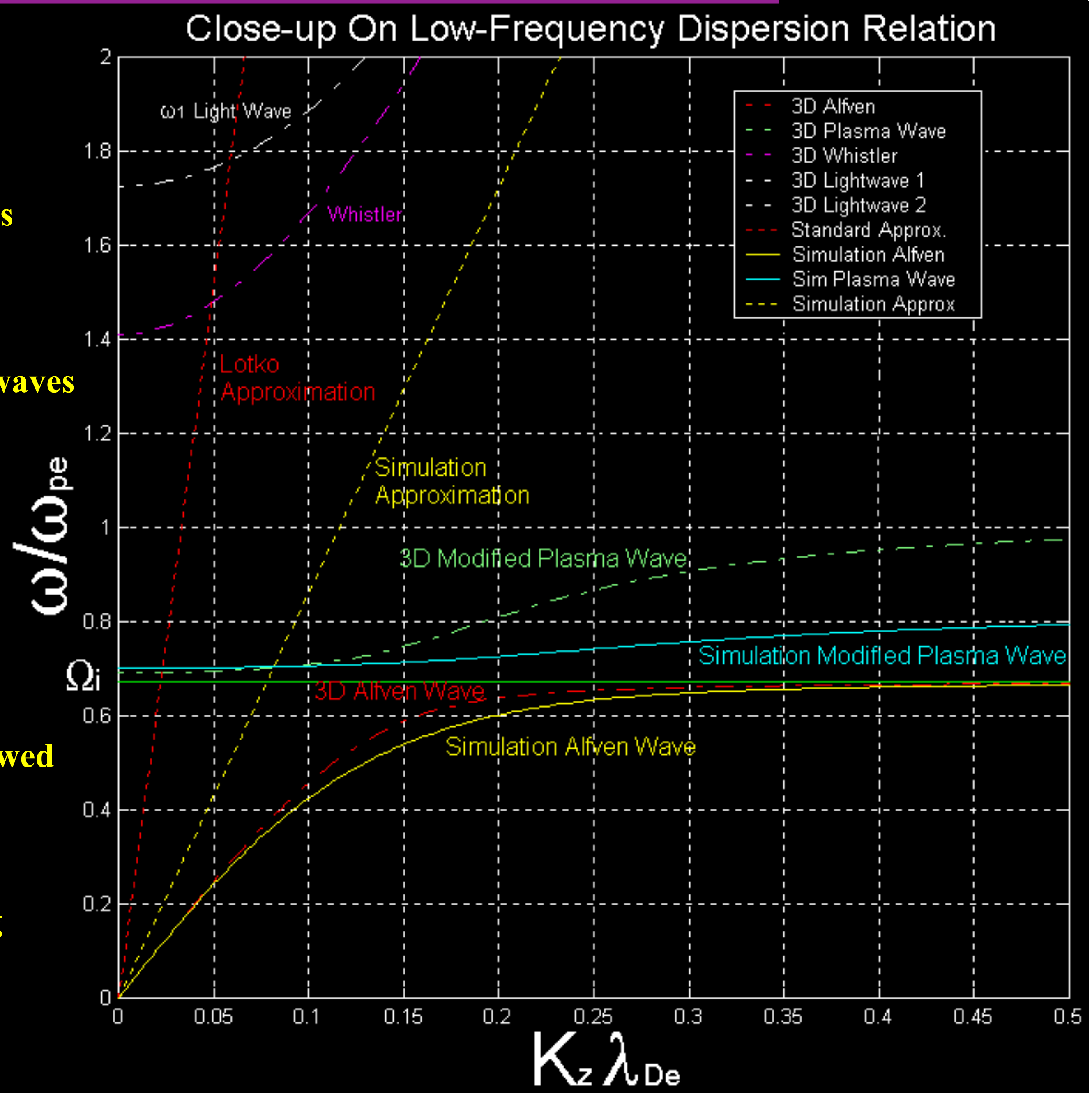
- Arrive at 6th order equation

- Taking limits
  W<<1 (low frequency)
  d<<1 (large ion mass)
  $K_{\parallel} \ll K_{\perp}$ (oblique propagation)

- Modified inertial Alfvén relation: $$W^2 = \left(\frac{1}{1+r^2}\right)\frac{K_{\parallel}^2 r^2}{1+K_{\perp}^2}$$

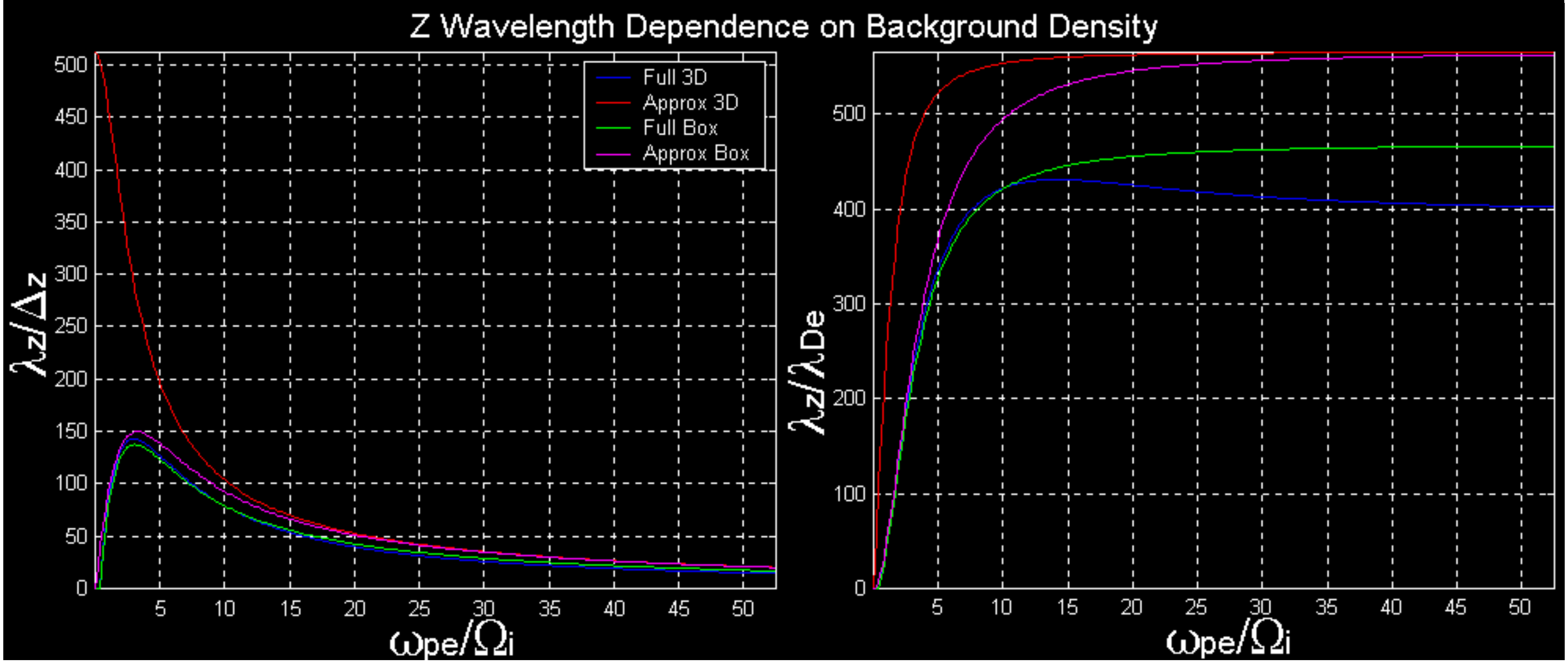
Alfvén Dispersion Graph

- Dashed lines are full 3D waves
- Solid Lines show simulation waves
- Simulation waves will have reduced parallel extent
- Phase velocity will also be slowed
- Not ordinary graph holds K[?] fixed while varying K[?]



Alfvén Gradient Implications

- **Increasing density can either increase or decrease the Alfvén wavelength**
- **Always increasing with regard to shrinking local skin depth**
- **Too highly magnetized plasma yields odd, increasing wavelengths with increasing density**



Alfvén Launcher Constraints

• Nine constraints were found which, if not satisfied, ruin an Alfvén run

I, II, III, IV) Memory available, minimum PPC, cell size, density contrast

Basic restriction to resolve Debye length, limit particle noise, and fit into memory

Failure makes run impossible, numerically unstable, or too noisy to interpret

V, VI) Resolve the skin depth (4 Debye lengths), avoid Landau damping

Failure results in no Alfvén wave produced

VII, IIX) Fit Alfvén wavelength in box, fit propagation cone in box

Failure results in too little Alfvén structure to study

IX) Ensure decreasing wavelength with increasing density

Failure does not well model auroral regions

• Everything from V on was discovered empirically requiring months of research

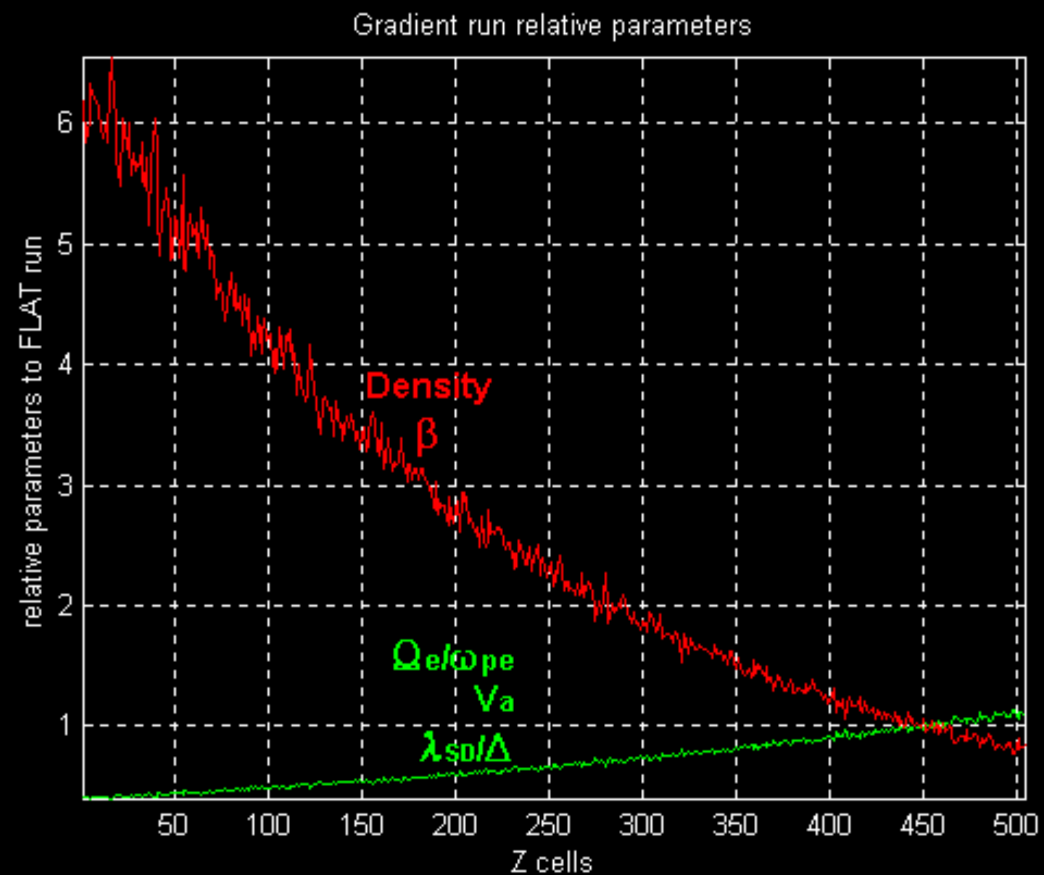
• We cannot satisfy IX from lack of RAM, and others only allow 1 parameter set

Alfvén Simulation Parameters

Flat Case

- Δ = $0.8\ \lambda_{De}$
- β = 0.00009
- Ω_e/ω_{pe} = 16.8
- Ω_i/ω_{pi} = 3.36
- $\lambda_{SD}/\ \lambda_{De}$ = 9
- $\lambda_{SD}/\ \Delta$ = 11.3
- V_A/c = 3.36
- m/M = 0.04 (1/25)
- T_i/T_e = 1
- $\Delta t \cdot \omega_{pe}$ = 0.12
- $\omega_{Launcher}$ = ½ Ω_i
- $\lambda_{Launcher}$ = 1/5 box
  = 51 Δ
  = 4.5 λ_{SD}

Gradient Case

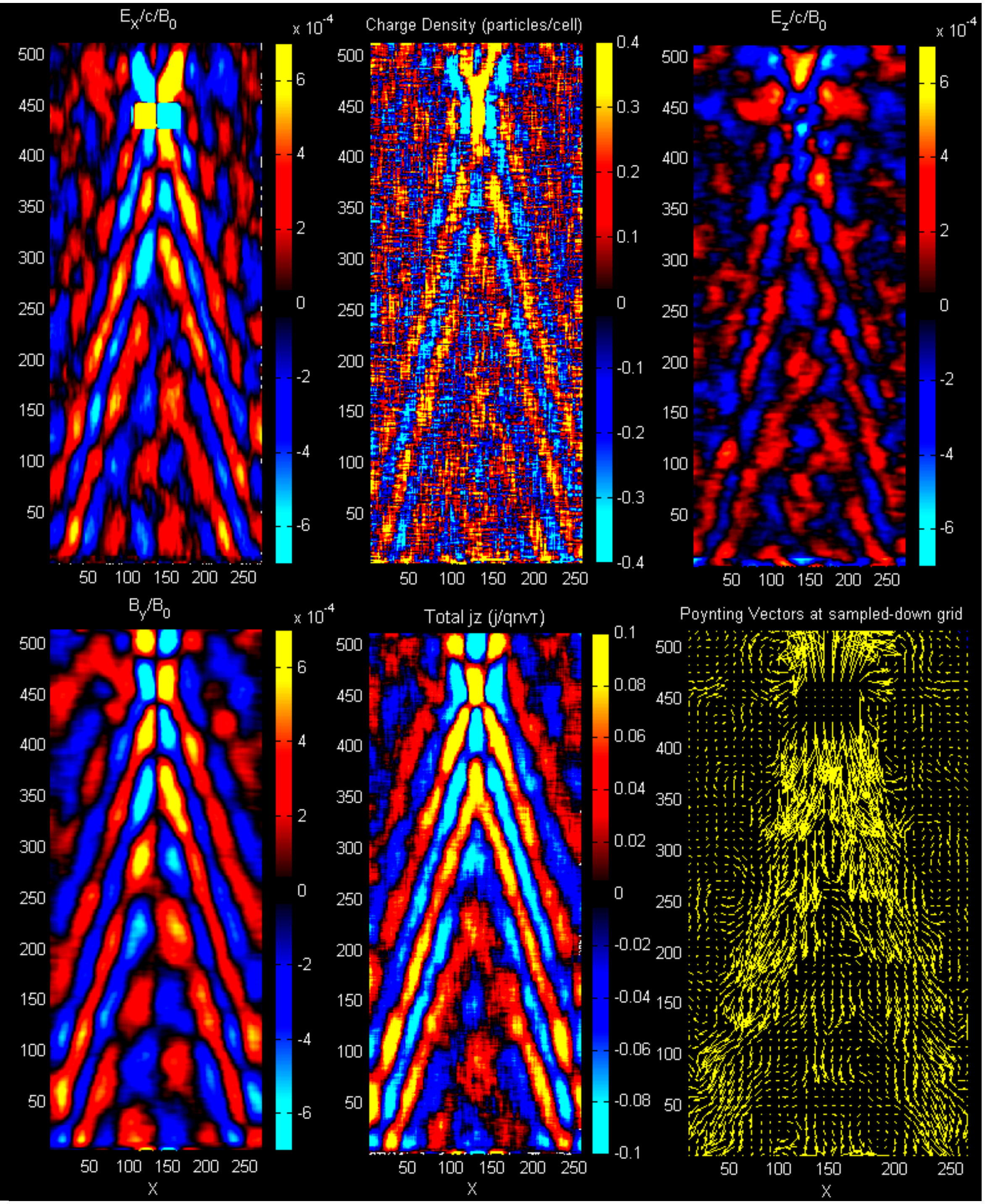


Flat Alfvén Launch

Movie

- Wave energy spread through several modes
- Some are in cones, others are collimated
- Parallel E field is around 1/8th perpendicular
- No boundary reflection observed
- No electromagnetic reflection observed
- Cone $\lambda_z = 100 \pm 6\Delta$ $v_z=0.49$
  Central $\lambda_z = 136 \pm 5\Delta$ $v_z=0.65$

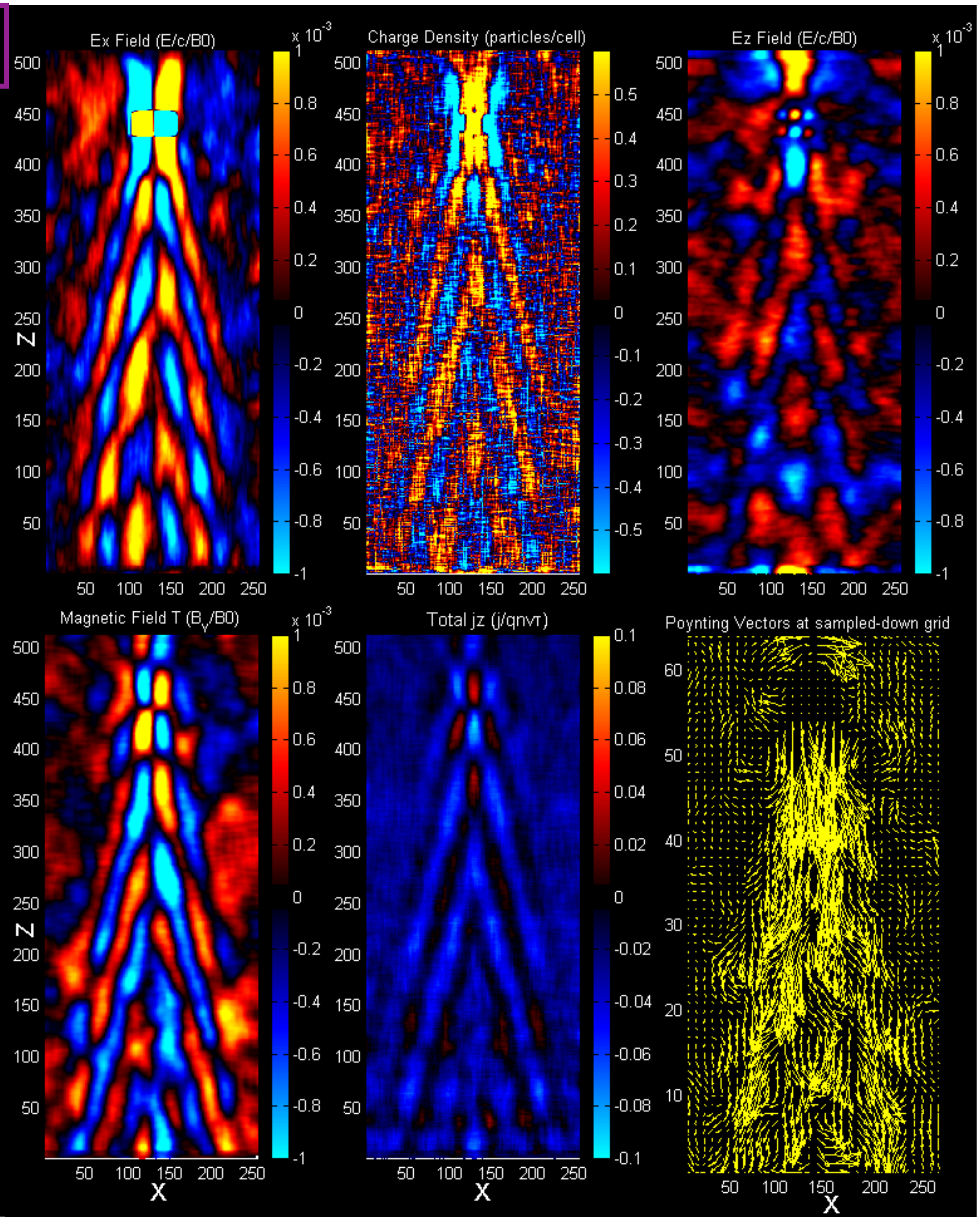
	mode 3	mode 4	mode 5	mode 6
Lz/cell	130	120	110	96
Vz/c	0.62	0.57	0.51	0.45
Cone	no	yes	yes	yes



Grad Alfvén Launch

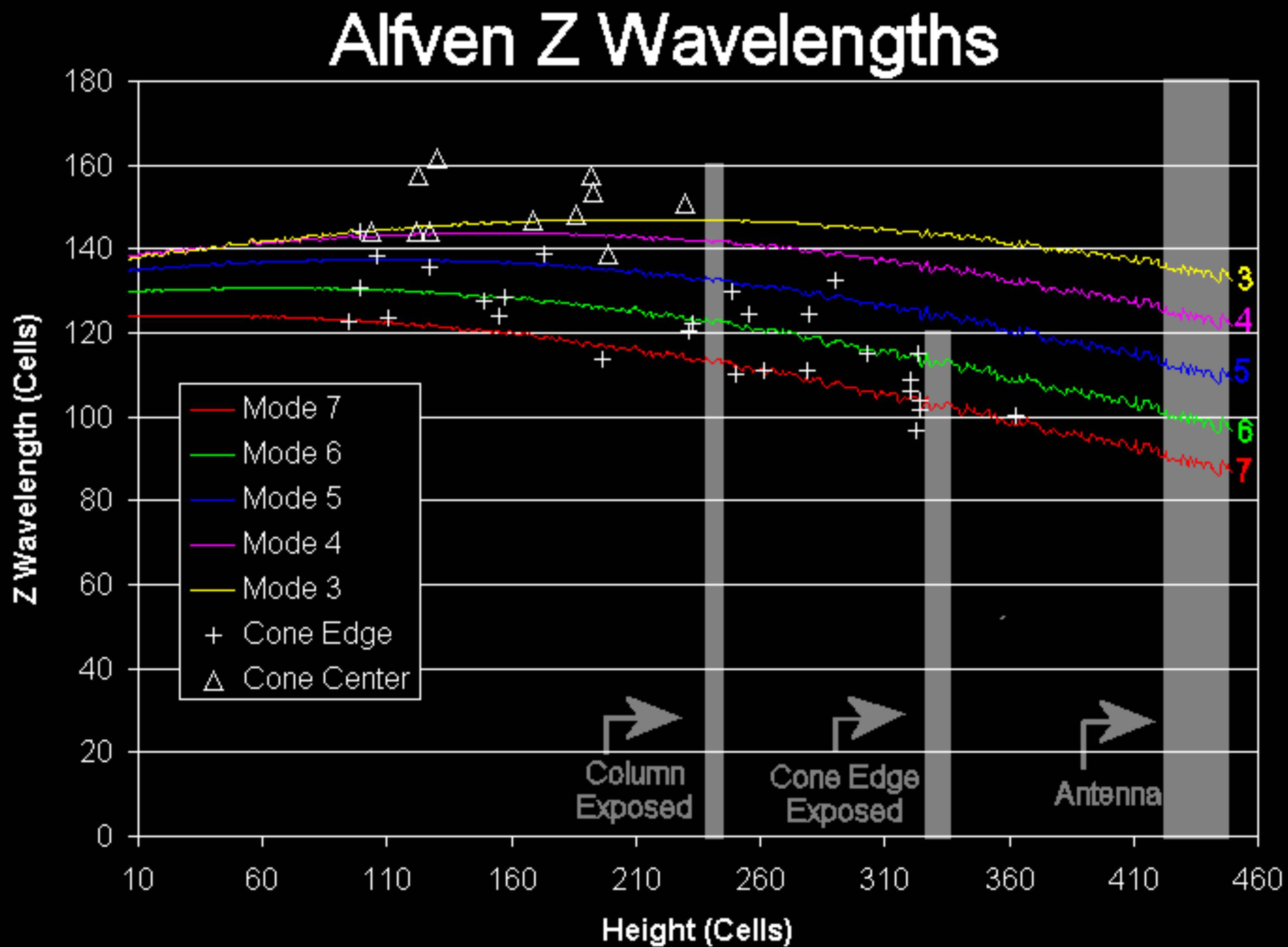
Movie 1 Movie2

- Elongation of λ_z alters angle of propagation
- 50% efficient wave reflection, Higher for center
- Comparison of wave characteristics more complex



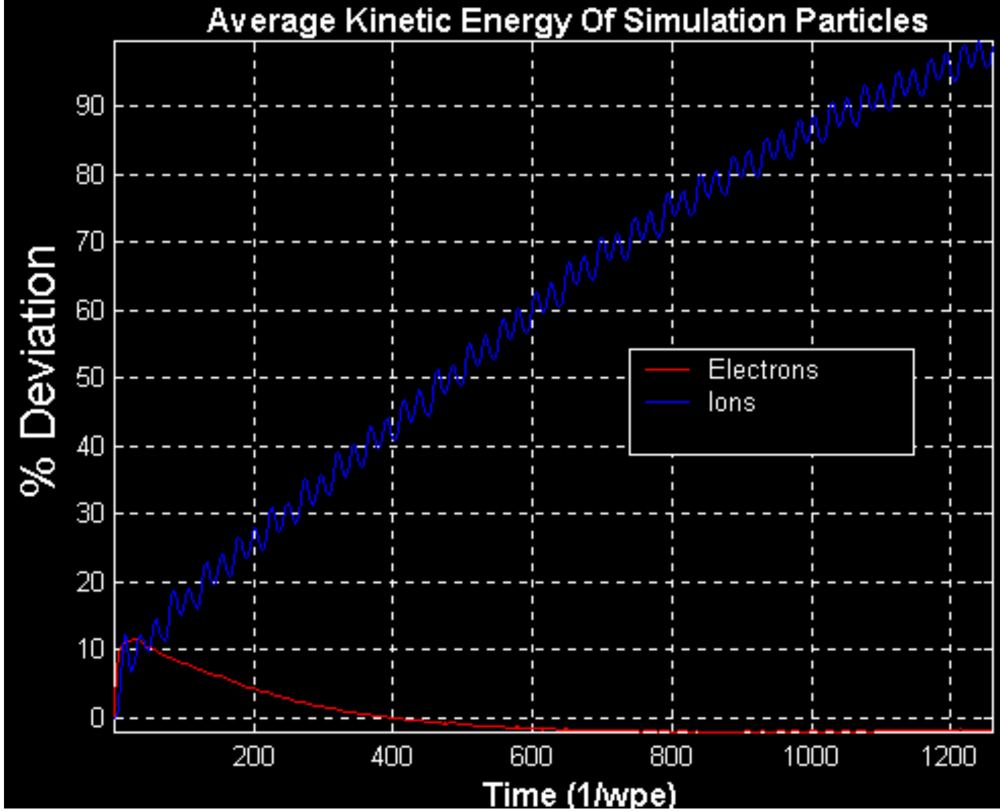
Alfvén Wavelength Modification

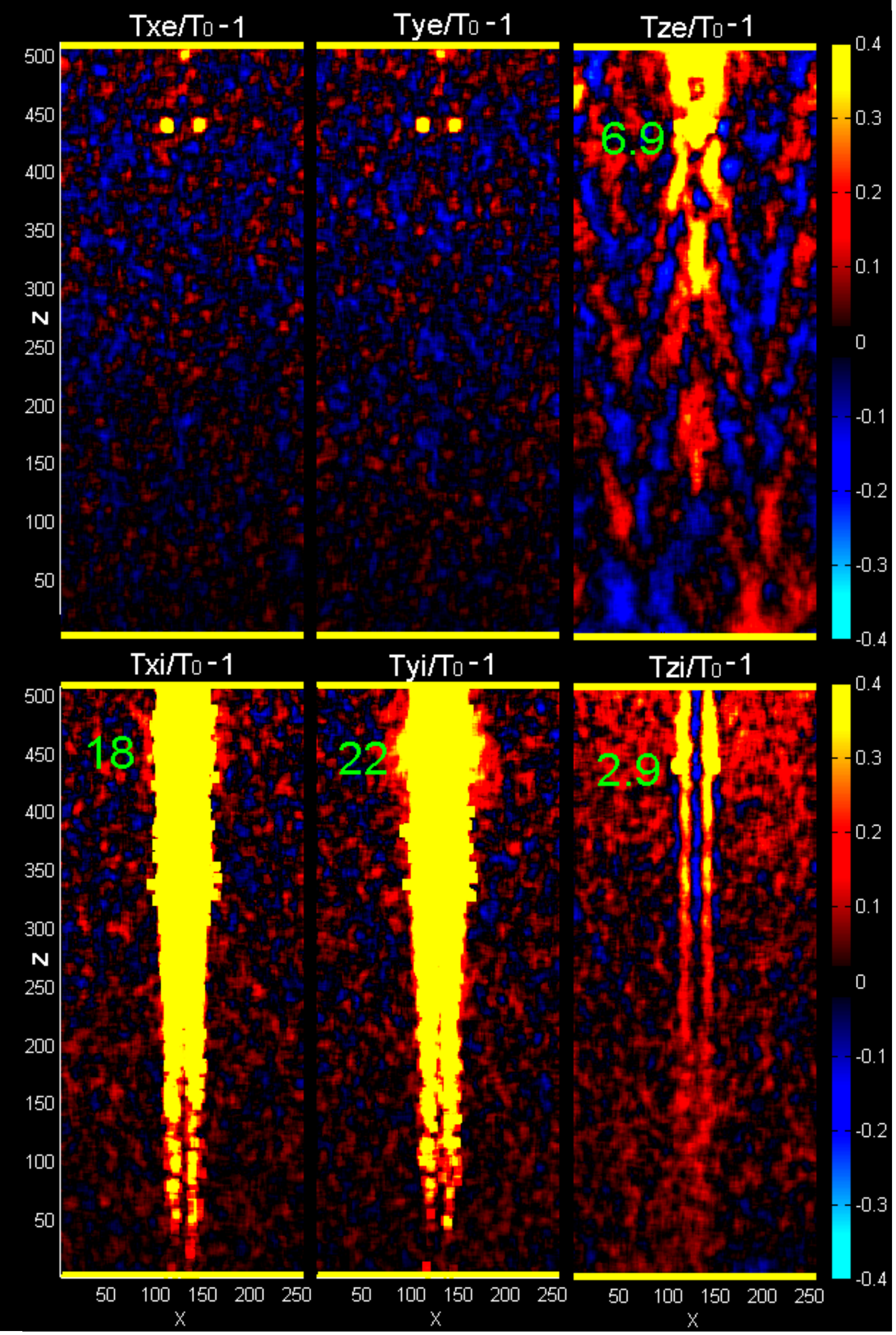
- Wavelength modification corresponds with linear theory background density alteration



Plasma Heating

- Ion are heated enormously by Alfvén launcher
- Unfortunately, all energy is direct ion emission from antenna
- No evidence for Alfvén flux heating the background is evident
- Most intense Alfvén flux is beneath drifting ions





Conclusions

Solitary Structure Study

- Finite beams generate robust electron holes
- Convective beam supports different non-linear mechanisms than absolute instabilities
- Parallel gradient extends lifetime of structures
- Clumping may explain prevalence of fast structures in down current region vs. up
- New beam-generation mechanisms were found to be associated with vortex interaction events
- Future work should include Neumann conditions at boundaries to study parallel potential drop evolution
- New gradient-ward acceleration remains mysterious

Alfvén Wave Study

- Computer limitations prevented completion
- Wavelength modification was observed
- Reflection instead of absorption observed
- More realistic parameters might reverse the above
- Requires about 300 Gig RAM to address completely

Code Study

- Code model is adept at addressing parallel gradient simulations
- Both electromagnetic and electrostatic versions were successful
- With additional RAM, this simulation should be used again in future

FAST Results

- Suggested mechanism from UP run on fast solitary wave isolation
- Even in upward current region, density profile along field lines is unknown… UP right might still apply
- Even FLAT run showed density spontaneous gradient formation
- Small-scale gradients may be more common and important